5

A Policy Strategy to Widen Opportunities to Learn Mathematics

Only a minority of the 15-year-old students in most countries understand and know well the core mathematics concepts in the curriculum. This chapter discusses a policy strategy to give all students similar opportunities to learn mathematics. Policy makers, curriculum designers, teachers and parents have an important role to play in the implementation of this strategy.

More than ever before, today's students need to understand mathematical ideas, compute fluently, engage in logical reasoning and communicate using mathematics. All these skills are central to a young person's preparedness to tackle problems that arise at work and in life beyond the classroom. But the reality is that many students are not reaching baseline levels of proficiency in mathematics (OECD, 2014; OECD, 2016). Large numbers of students also lack self-confidence in the subject, do not enjoy it and are unlikely to continue studying it voluntarily (Chapter 4).

How can these patterns be changed? This report shows that one possible way forward is to ensure that more students spend more "engaged" time learning mathematics concepts and practising complex mathematics tasks. The opportunity to learn mathematics content – the time students spend learning mathematics topics and practising mathematics tasks at school – can accurately predict mathematics literacy (Chapter 3). In the vast majority of countries, a substantial share of the performance disparities in PISA between socio-economically advantaged and disadvantaged students can be attributed to differences in these students' familiarity with mathematics concepts (Figure 3.16). Widening access to mathematics content can thus raise the average levels of achievement and, at the same time, reduce inequalities. While acquiring deep conceptual knowledge and procedural fluency, students should also have opportunities to practice their reasoning and modelling capabilities, and to develop positive attitudes and self-beliefs towards mathematics.

A policy strategy centred on giving all students similar opportunities to learn mathematics can reduce the number of students who lack the knowledge and understanding of mathematics expected of a 15-year-old. It can also encourage teachers to create and use engaging material to develop students' interest in mathematics. Ultimately, it can result in greater equity throughout the school system and thus in greater social mobility. Table 5.1 presents six policy recommendations that are part of this strategy.

DEVELOP COHERENT STANDARDS, FRAMEWORKS AND INSTRUCTION MATERIAL FOR ALL STUDENTS

Chapter 1 shows that only a minority of the 15-year-old students in most countries reported that they understand and know well the core mathematics concepts in the curriculum. For example, on average across OECD countries, less than 30% of students reported that they know well and understand the concept of arithmetic mean. Some students were never taught the topics, and many more were exposed to the topics so superficially that they only remember bits and pieces. Closing the gap between the expectations and the reality of mathematics learning is possible if education systems set the right objectives, and have the means to transform these objectives into everyday teaching and learning.

A first important step is to develop curriculum standards and frameworks that clearly define the mathematics content to be covered at each cycle of schooling and the targets to be reached by the end of compulsory schooling. Standards represent potential opportunities to learn (Schmidt and Burroughs, 2013) and define what students are expected to understand and be able to do. A framework is a more detailed list of the content and performance standards, by

grade level, that guides the development of the curriculum and the selection of instructional materials.

■ Table 5.1 ■
Policy recommendations to widen opportunities to learn

Policy recommendation	Goal	Who should be involved
Develop coherent standards, frameworks and instruction material for all students	▪ Reduce the number of students who have only superficial knowledge of core mathematics ideas ▪ Set high expectations for all students ▪ Set accountability targets while preserving teachers' autonomy	▪ Education policy makers, curriculum designers, teachers and those involved in the development of textbooks, assessments and teaching material
Help students acquire mathematics skills beyond content knowledge	▪ Strengthen the connections between school mathematics and the mathematics skills students will need for their future ▪ Reduce inequalities in quantitative skills ▪ Make school mathematics more engaging for all students	▪ Curriculum designers, teachers and those involved in the development of textbooks, assessments and teaching material
Reduce the impact of tracking on equity in exposure to mathematics	▪ Reduce the impact of socio-economic status on students' opportunities to learn ▪ Ensure that students in vocational tracks are exposed to a coherent curriculum	▪ Education policy makers
Learn how to handle heterogeneity in the classroom	▪ Give socio-economically disadvantaged students the same opportunities to learn as advantaged students ▪ Provide high-performing students with more challenging material	▪ Education policy makers and teachers
Support positive attitudes towards mathematics through innovations in curriculum and teaching	▪ Improve students' self-concept ▪ Reduce students' anxiety ▪ Foster motivation to learn mathematics, also after school hours	▪ Education policy makers, curriculum designers, teachers and parents
Monitor and analyse opportunities to learn	▪ Better understand the obstacles teachers face in covering the curriculum ▪ Better understand the obstacles students face in learning specific material ▪ Identify the impact of curriculum changes on students' acquisition of knowledge and skills	▪ Education policy makers, in collaboration with the research community

Content standards and frameworks should set the stage for the development of a core mathematics curriculum that provides all students, no matter their ability or socio-economic status, with the mathematics foundation for quantitative literacy and for study at a higher level. Giving all students equal opportunities to learn depends on how the curriculum is implemented (which will

be discussed later); but applying the same standards to all students can be a first step to break the pattern of differentiation that is based on the false assumption that only some students will need a strong background in mathematics for their future. For example, policy efforts to standardise secondary school curricula in Scotland in the 1980s improved average test scores and reduced socio-economic inequalities in education (Gamoran, 1996). Standards do not set upper bounds in content coverage and expected performance, so schools and teachers can adapt them to challenge high-performing students.

The standards should cover the full cycle of education to allow for the widest possible range of students to participate in a rigorous course of study from the outset. No standard can fully reflect the variety of abilities and needs among students in any given classroom. However, standards can provide clear signposts along the way to higher education and career readiness for all students, and can help identify early on those students who need special support.

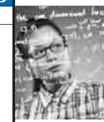
What content should be specified in the standards and frameworks? International comparisons find that the same set of core mathematics ideas is covered in the curricula of most PISA countries and economies (Table 1.3). This suggests that some consensus about what mathematics content is important may already have been reached (Mullis et al., 2012). What varies greatly across countries is how this core content is organised in the frameworks.

The curriculum can provide more productive exposure to mathematics content across all students if it is coherent, focused and not repetitive. Coherence can be achieved by grouping the concepts into units that, in their logical sequence, reflect the hierarchical nature of mathematics and make it easier for students to build on the skills they acquired (Schmidt et al., 2002).

When students understand the connections between the topics, they stop seeing mathematics as a laundry-list of formulas to memorise, and start to make sense of what they learn. A sequential or spiral curriculum framework is particularly useful for students who may not have been exposed to certain content. Before starting a new unit, teachers can easily identify what prior knowledge is needed, whether their students have had the opportunity to acquire this knowledge, and whether it is useful to review or incorporate previously studied topics in the new unit.

Coherence across topics and a strong focus on key mathematics ideas matter more than the quantity of topics covered. Tightening the focus of the curriculum by addressing fewer topics each year can allow for much greater depth of learning. For example, in Singapore the mathematics framework covers a relatively small number of topics in depth, following a spiral organisation in which topics introduced in one grade are covered in later grades, but at a more advanced level. Students are expected to have mastered prior content, not repeat it (Ginsburg et al., 2005). Korea cut as much as 20% of the learning content in the 2011 revision of the curriculum in order to let teachers cover mathematics topics in greater depth (Li and Lappan, 2013).

Curriculum frameworks that allocate sufficient time to a set of key topics can also leave more scope for teachers to choose how to cover the material and to adapt their teaching to their specific classroom context. Standards are meant to support teachers in their choices, and not to unnecessarily limit their autonomy. Teachers still have to use their professional judgement,



creativity and autonomy individually and together with other teachers to find the best ways to help their students to learn. For example, teachers in Singapore are not bound by the sequence of topics as long as they can ensure that the inherent hierarchy and linkages of mathematics are maintained (Ginsburg et al., 2005).

The effectiveness of standards to promote innovative patterns of instruction is clearly related to the development of tools and training, including teachers' guides, textbooks and pre-service education that prepare teachers to teach the curriculum and provide opportunities for professional development. Textbooks, in particular, can disrupt the coherence of the standards if they introduce concepts too early or too late, if they superficially cover the same concepts year after year, or fail to show students explicitly how the new material connects to previous concepts. More efforts at the national and international levels should thus be devoted to defining and regularly updating the core criteria that mathematics textbooks should meet to be eligible for use in the classroom.

HELP STUDENTS ACQUIRE MATHEMATICS SKILLS BEYOND CONTENT KNOWLEDGE

The mastery of core concepts and procedures is a necessary component of mathematics learning and is needed to understand and solve novel problems. However, knowledge of concepts or formulas and procedural fluency alone are hardly sufficient for solving complex problems (Chapter 3); several other skills are central to mathematics proficiency. These include the ability to use a wide range of mathematics strategies; the ability to reason using mathematical ideas and to communicate one's reasoning effectively; the ability to use the knowledge and time at one's disposal efficiently; and the disposition to see mathematics as sensible, useful and worthwhile, coupled with a belief in one's own efficacy (Schoenfeld, 2006; National Research Council, 2001).

Content, representations, tasks and teaching materials should be chosen and organised within and across grades to support the integrated and balanced development of all these abilities. Mathematics frameworks should thus be explicit about how concepts and skills are interwoven within individual units, over a year, and during the course of education.

Outside of their coursework, many students have difficulty doing what may be considered elementary mathematics for their level of attainment. For example, only 22% of the students tested in PISA 2012 were able to interpret a simple equation and explain the effect of a change to one variable on a second variable (the DRIP RATE problem, see Chapter 3). This probably happens because too many students spend too much time routinely and mechanically solving well-defined tasks that are very close to the ones they have been taught. These tasks do not involve exploration, conjecturing and thinking…in other words, they do not provide opportunities for deep learning. Similarly, simple applied tasks that are commonly used at school, such as "calculate how many square metres of tiles you need to cover a floor" (see Chapter 1), are routine mathematics tasks "dressed up" in the words of everyday life, and do not require any deep thinking and modelling skill (Echazarra et al., 2016). Chapter 3 shows that students' exposure to this type of applied task has only a modest relationship with students' capacity to solve PISA problems (Figure 3.8b).

Problem solving, as a method of teaching mathematics, can be used to introduce core mathematics concepts through lessons involving exploration and discovery (Stein et al., 2008). For example, the PISA 2012 REVOLVING DOOR item (see Chapter 3) describes a revolving door through diagrams. Students are asked to calculate the maximum arc length that each door can have so that air never flows between entrance and exit. Solving in class a problem like REVOLVING DOOR can help students consolidate their understanding of circle geometry and, at the same time, show them how they can test multiple heuristic strategies, such as dividing the circled space of the revolving door into six equal parts or reworking the diagram to consider extreme cases. Whatever strategy students choose to use, in order to succeed they need to monitor how well they are progressing, and persevere or change direction accordingly. Knowledge, effective reasoning, strategy formulation, self-regulation and perseverance can all be developed while working on problems of this type.

A greater emphasis on problem solving does not make traditional topics obsolete or irrelevant. The core mathematics topics in school curricula (fractions, functions, vectors…) are basic to all mathematics activities, including modelling and application. Whether one can solve mathematics problems depends in fundamental ways on the mathematics one knows. For example, students who do not know the formula for the circumference (π times diameter) would have a very difficult time solving complex, contextualised problems like REVOLVING DOOR or ARCHES (Chapter 3).

But introducing problem-solving strategies – such as teaching students how to question, make connections and predictions, conceptualise, and model complex problems – requires time and probably some adjustment to content coverage. Chapter 2 shows that cognitive-activation teaching strategies, such as giving problems with no immediate solution, might be associated with less content coverage in disadvantaged learning environments (Figure 2.23b).

When teachers develop a routine that allows students to discover and work with mathematics, teachers also need to consider how students with different skills approach complex problems. Chapter 3 shows that disadvantaged students perform more poorly than other students in those tasks requiring modelling skills (Figure 3.17). Problem solving, modelling and application make mathematics lessons more demanding, both for teachers and for students. Weaker students – and particularly those from a disadvantaged background – are less confident and tend to prefer more external direction (Lubienski and Stilwell, 1998). These students might need additional support in, for example, identifying the intended mathematical ideas embedded in contextualised problems, or describing those ideas to the rest of the class. That said, mathematics teachers should not be discouraged from integrating problem solving in their instruction when teaching weaker classes. Students with less familiarity with mathematics can still participate if the teacher builds a supportive relationship with students, conducts individualised tutoring sessions, builds on what students know, preserves equity among students in the classroom and makes explicit the desired classroom norms (Lester, 2007; Boaler, 2002; Lubienski, 2002). Formal and informal teacher networks can be useful platforms for sharing experiences and ideas.

Restructured textbooks, teaching materials and dedicated training can help minimise the time needed to incorporate these teaching practices into an already full schedule. For example, the New Zealand government's website on mathematics education (http://nzmaths.co.nz/)

provides teaching materials on problem solving, such as sample problems that fit into a given curriculum unit. The Mathematics Assessment Project, a collaboration between the University of California at Berkeley and Nottingham University, developed a series of "formative-assessment lessons" whose purpose is to help teachers improve students' ability to apply their knowledge to non-routine problems. The lessons describe common patterns of student responses to the tasks and ways to deal with them; they also contain activities that help teachers assess students' understanding and build on it.[1]

Mathematics teachers would also find it easier to integrate problem solving in instruction if the assessment system reflected the value of this approach. In reality, few curricula make the ability to model and apply mathematics knowledge the object of systematic assessment and testing (Rosli et al., 2013). More efforts need to be invested in constructing, using and sharing new ways to assess mathematics problem-solving abilities.

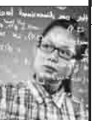
REDUCE THE IMPACT OF TRACKING ON EQUITY IN EXPOSURE TO MATHEMATICS

Access to mathematics is unequally distributed across individuals, schools and school systems, and familiarity with mathematics is strongly related to students' socio-economic status (Chapter 2). Moreover, the organisation of most education systems tends to reinforce socio-economic inequalities in access to mathematics (Chapter 2). Selecting student into more homogenous groups through grade repetition, between-school tracking, academic admission requirements and school transfers is associated not only with a more unequal distribution of achievement, but also with more unequal access to mathematics content, which is the basis for improving mathematics literacy.

Students who are enrolled in vocational tracks are much more likely to come from disadvantaged families and to have low levels of familiarity with mathematics (Figure 2.16). The concentration of disadvantaged students in less challenging tracks strengthens the link between socio-economic status and opportunities to learn, not only because students in vocational programmes are unlikely to receive the same exposure to mathematics as students in academic tracks, but also because students' outcomes and attitudes towards mathematics are affected by their peers (Chapter 4; Field, Kuczera and Pont, 2007).

Moreover, the starting age of between-school tracking at the system level is strongly related to equity in opportunities to learn mathematics (Figure 2.15). In systems that start sorting students into different tracks as early as 10 or 11, the relationship between students' socio-economic status and their access to mathematics at age 15 is much stronger than in systems where students are first tracked at age 15 or 16.

Postponing the age at which students are first sorted into tracks can be difficult, given the costs involved in such a substantial reform, possible effects on drop-out rates, and the reluctance among teachers who may have to adjust their teaching methods to cater to a more heterogeneous group of students. But several countries have been successful in delaying the age at first tracking, and there is some evidence that this policy has reduced the gap in education and labour market outcomes related to socio-economic status (Meghir and Palme, 2005; Hanushek



and Woessmann, 2006). In Sweden and Finland, which reformed their education systems between the 1950s and the 1970s, a later age at tracking reduced inequalities in the labour market later on (Meghir and Palme, 2005; Pekkarinen, Uusitalo and Kerr, 2009; Pekkala Kerr, Pekkarinen and Uusitalo, 2013). More recently, Germany reformed its education structure to reduce the influence of socio-economic status on student achievement. Some regions delayed the assignment of students to different tracks until they were 12 rather than 10 years old; some regions chose to combine tracks (going from a three- to a two-track lower secondary system); and some regions increased their system's flexibility by allowing students in any of the three types of lower secondary school to go to any type of upper secondary school (OECD, 2011a). In 1999, Poland reformed the structure of its education system, deferring tracking in secondary education, embracing deep curriculum reform, and giving more autonomy to schools. Research has shown that the deferral of tracking contributed to the substantial improvement in international assessments (OECD, 2011b).

It may not be necessary to eliminate early tracking as long as the education system provides students with equal opportunities to learn. In the Czech Republic and Singapore, for example, where students are first tracked at age 11 or 12, students' and schools' socio-economic profile explains less than 10% of the variation in familiarity with mathematics, similar to the OECD average (Figure 2.15). In other words, students in vocational tracks should be exposed to the same core curriculum and to the same quality of mathematics teaching as other students, and the tracking system should be flexible enough to allow students to change tracks when and if they are ready to do so. Moving towards equivalence in mathematics instruction between pathways would ensure that students can choose their preferred course of study and be confident they will acquire the core skills they need for their adult life.

LEARN HOW TO HANDLE HETEROGENEITY IN THE CLASSROOM

Given the difficulty of delaying early tracking, some countries have replaced between-school tracking with ability grouping within schools or within classrooms. Selection for within-school grouping may be better informed, since students' abilities are more easily observed in individual schools; but within-school or within-class ability grouping reduces opportunities to learn for disadvantaged students just as tracking does. In fact, the relationship between socio-economic status and mathematics achievement is not necessarily weaker in systems that use ability grouping compared with systems that use between-school tracking (Chmielewski, 2014). The negative impact of early tracking, streaming and grouping by ability on equity of education outcomes can be mitigated by limiting the number of subjects or the duration of ability grouping and by increasing opportunities to change tracks or classrooms (OECD, 2012).

The real alternative to streaming and ability grouping is heterogeneous classes. Teaching these classes can be challenging, and education authorities may have to provide additional assistance, such as more personalised tutoring and/or more innovative teaching practices, to the students who would otherwise be placed in "low" tracks. Schools with mixed-ability classes must also avoid lowering academic standards and must provide their high-achievers with challenging material (Gamoran, 1996; Gamoran, 2002).

Teach heterogeneous classes effectively

It is easier and more efficient for teachers to deal with a smaller range of abilities than to provide instruction that is sufficiently broad to address the needs of all students or to "teach to the middle" (Darrow, 2003; Evertson, Sanford and Emmer, 1981). According to PISA results, many teachers believe that heterogeneous classes hinder learning, especially in socio-economically disadvantaged schools (Figure 2.17). Clearly, teachers need to adapt their teaching strategies depending not only on the average ability of the class but also on the degree of ability heterogeneity. For instance, Chapter 2 shows that some teaching practices, such as cognitive-activation strategies, are related in different ways to familiarity with mathematics and performance in mathematics across schools with different socio-economic profiles (Figure 2.23a and 2.23b).

Nevertheless, there are ways to help teachers work with heterogeneous classes other than by sorting students. As was mentioned above, curricula can be organised in a spiral, so that they cover key ideas several times, in order to provide students with multiple opportunities to learn important concepts at varying levels of complexity. In addition, classes can be made smaller as a way to make it less difficult to teach heterogeneous groups (Finland did this in the 1980s when it discontinued ability grouping [Kupari, 2008]).

Moreover, specific teaching practices, such as using curricula and pedagogies with multiple points of entry that are challenging, relevant and engaging, can be adopted in the context of whole-class instruction. Various student-oriented practices also appear to be successful in heterogeneous class environments, such as having students work in small (heterogeneous) groups, co-operative learning, keeping students actively involved and giving students control over their learning (Freedman, Delp and Crawford, 2005; Rubin, 2006). In particular, flexible grouping can be frequently reconfigured based on content, project and ongoing evaluation, as a way to nurture the idea that ability is not fixed, and to reduce the segregation that often comes with more rigid forms of grouping (Tomlinson, 2001). Co-operative learning strategies are also used in heterogeneous classes (Rothenberg, Mcdermott and Martin, 1998). Success for All, for example, is a programme for primary schools in the United States and the United Kingdom that combines co-operative learning with small ability groups that are frequently reorganised to reflect student progress (Loveless, 2016).

Given the crucial role of teachers in implementing curricula in challenging contexts, teachers who shift from teaching classes grouped by ability to teaching heterogeneous classes, as well as teachers addressing students in multicultural and diverse environments, should receive support and training on how to teach such classes (OECD, 2010; Rubin and Noguera, 2004).

Offer greater and individualised support to struggling students

Struggling students should receive instruction adapted to their ability and needs. For many low-performing students, this may include simply *more time* to learn. Engaged learning time is a key aspect of opportunity to learn, in addition to the curriculum content to which students are exposed. Longer instruction time is related to better performance – except when that learning time is wasted in a disruptive disciplinary climate (Chapter 3). The so-called "No Excuses" charter

schools in the United States provide an example of how combining dramatically increased instructional time, strict behaviour norms and a strong student work ethic has improved student achievement in low-income, minority and disadvantaged schools (Angrist et al., 2010; Thernstrom and Thernstrom, 2004).

In addition to more instruction, many disadvantaged students also need more tailored approaches and individualised support. In the context of the US debate on "de-tracking" (intended as the shift from ability grouping to mixed-ability, heterogeneous classes), targeted academic support is considered a key practice to be embedded in the regular organisation of the school year or day. Support classes can help struggling students catch up on skills and concepts they may have missed in the classroom, and can support them in completing their daily work, without preventing them from being exposed to a more academically demanding curriculum (Rubin, 2006). High-dose, targeted tutoring is one of several practices that have been shown to improve achievement in low-performing US charter schools (Dobbie and Fryer, 2013; Fryer, 2011). Of course, the effectiveness of this remedial support depends on the quality of its implementation, and particularly on teachers' preparedness.

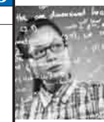
Finland, for example, provides dedicated support to students in need. Half of children with special education needs are mainstreamed, rather than being enrolled in special schools. These students are helped by "special teachers" assigned to each school, based on the idea that if schools focus on early diagnosis and intervention, most students can be helped to achieve success in regular classrooms. These specially trained teachers work closely with the class teachers to identify students in need of extra help and to work individually or in small groups with these students so that they get the support they need to keep up with their classmates (OECD, 2011a).

Low-performing students in Singapore are given extra help from well-trained teachers and follow an alternative mathematics framework that covers all the mathematics topics in the regular framework, but at a slower pace and with more repetition (Ginsburg et al., 2005).

SUPPORT POSITIVE ATTITUDES TOWARDS MATHEMATICS THROUGH INNOVATIONS IN CURRICULA AND TEACHING

The types of mathematics that students are exposed to are related not only to their achievement but also to their attitudes and self-beliefs about mathematics (Chapter 4). Exposure to relatively complex mathematics topics may improve the attitudes and self-beliefs of students who are relatively well-prepared and ready to be challenged, but it may undermine the self-beliefs of students who do not feel up to the tasks. In Belgium, Denmark, Macao-China, the Netherlands and Switzerland, students of similar ability who are more exposed to pure mathematic tasks tend to display lower mathematics self-concept than students who are less exposed (Table 4.6). On average across OECD countries, greater exposure to complex concepts, as measured by familiarity with mathematics, is associated with less mathematics anxiety among high-performing students but with greater anxiety among low-performing students (Figure 4.8).

Given the link between attitudes towards mathematics and performance (OECD, 2013), in designing or revising their mathematics curricula, countries should find ways of improving performance and problem-solving skills without undermining the self-confidence or raising



the anxiety of low-performing students. Box 4.2 offers some examples from Australia, Hong Kong-China, Korea and Singapore of how the development of positive attitudes towards mathematics became an aim of the mathematics curriculum, and of how some of these countries and economies have reduced curriculum content to give more time to engaging activities that would improve students' motivation.

Teaching practices can play a big role in influencing students' attitudes towards learning. Chapter 4 discusses how various teaching practices – including feedback and communication, cognitive-activation strategies and student-oriented practices – are associated with students' attitudes and self-beliefs towards mathematics, even though the extent of the relationship varies according to students' familiarity with mathematics. Reinforcing basic numerical and spatial skills, reducing time pressure during tests, and bolstering teachers' ability and confidence to teach mathematics can also be ways of reducing students' mathematics anxiety (Maloney and Beilock, 2012; Beilock and Willingham, 2014).

However, it is also important to take into account that teaching practices and curriculum coverage may interact in affecting students' self-beliefs, especially among students who have had fewer opportunities to learn. Teaching practices aimed to reinforce students' self-beliefs and improve their attitudes towards learning may need to be adapted to students' readiness to learn the mathematics specified in the curriculum.

Teachers can also help to improve students' attitudes towards mathematics by engaging parents. Chapter 4 has also shown that parents are likely to transmit their mathematics anxiety to their children, especially through their involvement in homework. Teachers can make parents aware of their influence and of the importance of communicating positive messages about mathematics, and can suggest alternative ways of supporting their children in learning mathematics (such as external tutors or after-school classes).

MONITOR AND ANALYSE OPPORTUNITIES TO LEARN

The content of instruction, as defined in the curriculum, plays a fundamental role in students' mathematics achievement (Chapter 3; Gamoran, 2007; Schmidt et al., 2015). However, some would argue that what matters more is how a curriculum is implemented rather than the curriculum itself (Chapter 1). In that case, data collection and analysis must try to determine how well teachers cover the curriculum and whether the implemented curriculum adheres to the objectives set in the standards.

Measuring opportunities to learn involves more than a simple time metric. Data need to be collected all along the progression from what educators and policy makers specify in the standards, through the content to which students are exposed, to the ideas and practices that students understand (Floden, 2002). An analysis of data at each step of the progression can help clarify how much time teachers spend teaching each topic in the curriculum, how many items in a national assessment are devoted to the topic, and the degree to which students engage in the corresponding instructional activities.

Monitoring opportunities to learn thus requires combining multiple data-collection instruments and analyses. A core set of information should be collected directly from teachers, using either surveys questions that ask teachers about the content covered in their classes (such as those used in the Trends in International Mathematics and Science Study [TIMSS]), or teacher logs completed over the course of the school year. After comparing multiple indicators, Gamoran et al. (1997) concluded that the measure of OTL that more strongly correlates with students' achievement is an index that combines teacher-reported information about the proportion of class time spent on topics with information on the extent to which the instruction engaged students' problem-solving abilities. Collecting and processing log data and questionnaires can be made less cumbersome and costly by adapting digital approaches from other types of data collections, such as time-use studies.

Students' self-reported knowledge and exposure to mathematics tasks, as collected in PISA (see Chapter 1), provides another piece of information about what students actually get out of mathematics instruction. Collating the information on the intended curriculum (from official documents), on the implemented curriculum (from teachers' reports and textbooks), and on what students know and can do in class (from students' self-reports) can give a detailed picture of any missing steps in the progression from curriculum intentions to student outcomes.

All these data are of little value if they are not properly used to guide changes in the curriculum. In some countries and economies, including France, Hong Kong-China, the Netherlands, Singapore, the United Kingdom and the United States, national centres conduct multi-year research and curriculum-development programmes in school mathematics. These institutes and universities should engage in rigorous statistical comparisons of student performance under "traditional" and "experimental" curricula (where performance is evaluated on the basis of carefully designed assessments of different mathematics skills). These evaluations help identify what innovative elements in the curriculum should be strengthened. Ideally, results disaggregated by characteristics of the school (such as its socio-economic profile) could provide evidence of the effects of the curriculum and the kinds of support structures that are helpful in different implementation contexts (Schoenfeld, 2006).

Students' learning depends not only on the content covered by the teacher but also on other dimensions of the learning environment, such as an orderly classroom climate and the pedagogical choices teachers make. For example, Chapter 3 shows that long instruction time is related to high performance only in classrooms with a good disciplinary climate (Figure 3.6). Video studies can provide data directly from the classroom about how teachers structure and manage their classes, what types of support and student orientation they adopt, and whether they use teaching methods and tasks that challenge students' cognitive abilities, self-beliefs and attitudes (Tomáš and Seidel, 2013). Video studies can also showcase how students acquire mathematics knowledge, skills and understanding, and thus inform the design and update of mathematics standards. The Teaching and Leaning International Survey (TALIS) study is piloting an international video study of teaching practices. The objective of the study is to provide insights into effective teaching practices using classroom observations from countries with different teaching cultures. The pilot will also eventually produce a global video library showcasing a variety of teaching practices in a range of educational settings in participating countries.[2]

Notes

1. The formative assessment tasks and more information on the Mathematics Assessment Project are available at: map.mathshell.org.

2. More information on the TALIS international video study is available at: https://www.oecd.org/edu/school/TALIS-2018-video-study-brochure-ENG.pdf.

References

Angrist, J.D., et al. (2010), "Inputs and impacts in charter schools: KIPP Lynn", *American Economic Review,* Vol. 100/2, pp. 239-43, http://dx.doi.org/10.1257/aer.100.2.239.

Beilock, S.L. and **D.T. Willingham** (2014), "Math anxiety: Can teachers help students reduce it?", *American Educator,* Vol. 38/2, pp. 28-32.

Boaler, J. (2002), "Learning from Teaching: Exploring the Relationship between Reform Curriculum and Equity", *Journal for Research in Mathematics Education,* Vol. 33/4, pp. 239-258, http://dx.doi.org/10.2307/749740.

Chmielewski, A.K. (2014), "An international comparison of achievement inequality in within- and between-school tracking systems", *American Journal of Education*, Vol. 120/3, pp. 293-324, http://dx.doi.org/doi:10.1086/675529.

Darrow, A.A. (2003), "Dealing with diversity: The inclusion of students with disabilities in music", *Research Studies in Music Education,* Vol. 21/1, pp. 45-57, http://dx.doi.org/10.1177/1321103X030210010401.

Dobbie, W. and **R. Fryer** (2013), "Getting beneath the veil of effective schools: Evidence from New York City", *American Economic Journal: Applied Economics,* Vol. 5/4, pp. 28-60.

Echazarra, A., et al. (2016), "How teachers teach and students learn: Successful strategies for school", *OECD Education Working Papers*, No. 130, OECD Publishing, Paris, http://dx.doi.org/10.1787/5jm29kpt0xxx-en

Evertson, C.M., J.P. Sanford and **E.T. Emmer** (1981), "Effects of class heterogeneity in junior high school", *American Educational Research Journal,* Vol. 18/2, pp. 219-232, http://dx.doi.org/10.3102/00028312018002219.

Field, S., M. Kuczera and **B. Pont** (2007), *No More Failures: Ten Steps to Equity in Education*, Education and Training Policy, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264032606-en.

Floden, R.E. (2002), "The measurement of opportunity to learn", in A.C. Porter and A. Gamoran (eds.), *Methodological Advances in Cross-National Surveys of Educational Achievement,* The National Academies Press, Washington, DC, http://www.nap.edu/read/10322/chapter/9.

Freedman, S.W., V. Delp and **S.M. Crawford** (2005), "Teaching English in untracked classrooms", *Research in the Teaching of English*, Vol. 40/1, pp. 62-126.

Fryer, R.G. (2011), "Injecting successful charter school strategies into traditional public schools: A field experiment in Houston", *NBER Working Paper*, No. 17494. National Bureau of Economic Research, Cambridge, MA.

Gamoran, A. (ed.) (2007), *Standards-Based Reform and the Poverty Gap*, Brookings Institution, Washington, DC.

Gamoran, A. (2002), "Standards, inequality & ability grouping in schools", CES Briefings, No. 25, Centre for Educational Sociology, University of Edinburg, http://www.ces.ed.ac.uk/PDF%20Files/Brief025.pdf.

Gamoran, A. (1996), "Curriculum standardization and equality of opportunity in Scottish secondary education: 1984-90", *Sociology of Education*, Vol. 69/1, pp. 1-21, http://dx.doi.org/doi:10.2307/2112720.

Gamoran, A., A.C. Porter, J. Smithson and **P.A. White** (1997), "Upgrading high school mathematics instruction: Improving learning opportunities for low-achieving, low-income youth", *Educational Evaluation and Policy Analysis*, Vol. 19/4, pp. 325-38, http://dx.doi.org/doi:10.3102/01623737019004325.

Ginsburg, A., S. Leinwand, T. Anstrom and **E. Pollock** (2005), *What the United States Can Learn From Singapore's World-class Mathematics System (and What Singapore Can Learn from the United States): An Exploratory Study*, American Institutes for Research, Washington, DC, http://www.air.org/resource/what-united-states-can-learn-singapore%E2%80%99s-world-class-mathematics-system-exploratory-study.

Hanushek, E.A. and **L. Woessmann** (2006), "Does educational tracking affect performance and inequality? Differences-in-differences evidence across countries", *The Economic Journal*, Vol. 116/510, pp. 63-76, http://dx.doi.org/doi:10.1111/j.1468-0297.2006.01076.x.

Kupari, P. (2008), "Mathematics education in Finnish comprehensive school: Characteristics contributing to student success", Proceedings of the 11th International Congress on Mathematical Education, Monterrey, Mexico, http://www.mathunion.org/fileadmin/ICMI/files/About_ICMI/Publications_about_ICMI/ICME_11/Kupari.pdf.

Lester, F.K. (ed.) (2007), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics*, Information Age Publishing, Charlotte, NC.

Li, Y. and **G. Lappan** (2013), *Mathematics Curriculum in School Education*, Springer, Berlin.

Loveless, T. (2016), "The resurgence of ability grouping and persistence of tracking", http://www.brookings.edu/research/reports/2013/03/18-tracking-ability-grouping-loveless (accessed 01 June 2016).

Lubienski, S.T. (2002), "Research, reform, and equity in U.S. mathematics education", *Mathematical Thinking and Learning*, Vol. 4/2-3, pp. 103-25, http://dx.doi.org/doi:10.1207/S15327833MTL04023_2.

Lubienski, S.T. and **J. Stilwell** (1998), "Teaching low-SES students mathematics through problem solving: Tough issues, promising strategies and lingering dilemmas", in H. Schoen and R.I. Charles (eds.), *Teaching Mathematics through Problem Solving: Grades 6-12*, National Council of Teachers of Mathematics, Reston, VA, pp. 207-218.

Maloney, E.A. and **S.L. Beilock** (2012), "Math anxiety: Who has it, why it develops, and how to guard against it", *Trends in Cognitive Sciences*, Vol. 16/8, pp. 404-406, http://dx.doi.org/doi:10.1016/j.tics.2012.06.008.

Meghir, C. and **M. Palme** (2005), "Educational reform, ability, and family background", *American Economic Review*, Vol. 95/1, pp. 414-24, http://dx.doi.org/doi:10.1257/0002828053828671.

Mullis, I.V.S. et al. (eds.) (2012), *TIMSS 2011 Encyclopedia: Education Policy and Curriculum in Mathematics and Science, Volumes 1 and 2*, TIMSS & PIRLS International Study Center, Boston College, Chestnut Hill, MA, http://timssandpirls.bc.edu/timss2011/encyclopedia-timss.html.

National Research Council (2001), *Adding It Up: Helping Children Learn Mathematics*, National Academies Press, Washington, DC, http://www.nap.edu/catalog/9822.

OECD (2016), *Low-Performing Students: Why They Fall Behind and How To Help Them Succeed*, PISA, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264250246-en.

OECD (2014), *PISA 2012 Results: What Students Know and Can Do (Volume I, Revised edition, February 2014): Student Performance in Mathematics, Reading and Science*, PISA, OECD Publishing, http://dx.doi.org/10.1787/9789264208780-en.

OECD (2013), *PISA 2012 Results: Ready to Learn (Volume III): Students' Engagement, Drive and Self-Beliefs*, PISA, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264201170-en.

OECD (2012), *Equity and Quality in Education: Supporting Disadvantaged Students and Schools*, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264130852-en.

OECD (2011a), *Lessons from PISA for the United States*, Strong Performers and Successful Reformers in Education, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264096660-en.

OECD (2011b), "The impact of the 1999 education reform in Poland", *OECD Education Working Papers*, No. 49, OECD Publishing, Paris, http://dx.doi.org/10.1787/5kmbjgkm1m9x-en.

OECD (2010), *Educating Teachers for Diversity: Meeting the Challenge*, Centre for Educational Research and Innovation, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264079731-en.

Pekkala Kerr, S., T. Pekkarinen and **R. Uusitalo** (2013), "School tracking and development of cognitive skills", *Journal of Labor Economics*, Vol. 31/3, pp. 577-602, http://dx.doi.org/doi:10.1086/669493.

Pekkarinen, T., R. Uusitalo and **S. Kerr** (2009), "School tracking and intergenerational income mobility: Evidence from the Finnish comprehensive school reform", *Journal of Public Economics*, Vol. 93/7-8, pp. 965-73, http://dx.doi.org/doi:10.1016/j.jpubeco.2009.04.006.

Rosli, R., D. Goldsby and **M.M. Capraro** (2013), "Assessing students' mathematical problem-solving and problem-posing skills", *Asian Social Science*, Vol. 9/16, pp. 54-60.

Rothenberg, J.J., P. Mcdermott and **G. Martin** (1998), "Changes in pedagogy: A qualitative result of teaching heterogeneous classes", *Teaching and Teacher Education*, Vol. 14/6, pp. 633-642, http://dx.doi.org/doi:10.1016/S0742-051X(98)00013-4.

Rubin, B.C. (2006), "Tracking and detracking: Debates, evidence, and best practices for a heterogeneous world", *Theory Into Practice*, Vol. 45/1, pp. 4-14, http://dx.doi.org/doi:10.1207/s15430421tip4501_2.

Rubin, B.C. and **P.A. Noguera** (2004), "Tracking detracking: Sorting through the dilemmas and possibilities of detracking in practice", *Equity & Excellence in Education*, Vol. 37/1, pp. 92-101, http://dx.doi.org/doi:10.1080/10665680490422142.

Schmidt, W.H. and **N.A. Burroughs** (2013), "Springing to life: How greater educational equality could grow from the common core mathematics standards", *American Educator*, Vol. 37/1, pp. 2-9.

Schmidt, W.H., N.A. Burroughs, P. Zoido and **R.T. Houang** (2015), "The role of schooling in perpetuating educational inequality an international perspective", *Educational Researcher*, Vol. 44/7, pp. 371-386, http://dx.doi.org/doi:10.3102/0013189X15603982.

Schmidt, W.H., R. Houang and **L. Cogan** (2002), "A coherent curriculum: The case of mathematics", *American Educator*, Vol. 26/2, pp. 10.

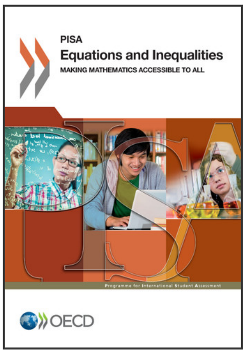
Schoenfeld, A.H. (2006), "What doesn't work: The challenge and failure of the What Works Clearinghouse to conduct meaningful reviews of studies of mathematics curricula", *Educational Researcher*, Vol. 35/2, pp. 13-21.

Stein, M.K., R.A. Engle, M.S. Smith and **E.K. Hughes** (2008), "Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell", *Mathematical Thinking and Learning*, Vol. 10/4, pp. 313-340, http://dx.doi.org/doi:10.1080/10986060802229675.

Thernstrom, A. and **S. Thernstrom** (2004), *No Excuses: Closing the Racial Gap in Learning*, Simon and Schuster, New York, NY.

Tomáš, J. and **T. Seidel** (2013), *The Power of Video Studies in Investigating Teaching and Learning in the Classroom*, Waxmann, Munster, Germany.

Tomlinson, C.A. (2001), *How to Differentiate Instruction in Mixed-Ability Classrooms*, Association for Supervision and Curriculum Development, Alexandria, VA.



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