3

Exposure to Mathematics in School and Performance in PISA

This chapter analyses how opportunity to learn mathematics influences students' performance in PISA and their capacity to solve the most challenging PISA tasks. The results show that exposure to pure mathematics has a strong association with performance that tends to increase as the difficulty of mathematics problems increases. Socio-economically disadvantaged students, who have fewer opportunities to learn how to use symbolic language, acquire fluency in procedures and build mathematic models, lack some of the essential skills needed to solve mathematics problems.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

It is hard to find two scholars holding the same view of how mathematics should be taught, but there is a general agreement among practitioners about the final goal: mathematics should be taught "as to be useful" (Freudenthal, 1968; Gardiner, 2004). In other words, mathematics should help students build competent and flexible performance on "demanding" tasks (Schoenfeld, 1994; Schoenfeld, 2004). Competent performance implies that mathematical operations are fast and effortless; flexible performance means not just solving familiar problems, but also being able to tackle novel problems built on the same principles (Rosenberg-Lee and Lovett, 2006).

Are mathematics curricula structured and implemented in ways that help students to develop competence and flexibility for demanding tasks? An analysis of PISA data can help to answer this question. PISA challenges students to solve a set of problems that might be encountered in real life, that are of greater or lesser difficulty and that do not look like those presented in mathematics classes at school. By analysing how students who have been exposed to mathematics to varying degrees perform on different PISA tasks, this chapter provides new evidence on whether students can apply the mathematics they learn at school.

While PISA data cannot establish cause and effect, the analysis shows a positive relationship between exposure to pure mathematics and performance. This is not only because "smarter" students may be concentrated in the same schools. Rather, students in the same school who are more frequently exposed to pure mathematics tend to do better in PISA. The chapter also looks at the relative strengths and weaknesses of countries and students, particularly disadvantaged students, across different areas of mathematics.

What the data tell us

- On average across OECD countries, student performance on mathematics tasks requiring familiarity with algebraic operations improved between 2003 and 2012, while performance on tasks with a focus on geometry deteriorated.
- In Austria, Croatia, Korea, Romania, Shanghai-China and Chinese Taipei, re-allocating one hour of instruction from reading to mathematics is associated with an improvement in mathematics performance, compared with reading performance, of more than 10 score points. However, the effect of such changes in instruction time on performance is not statistically significant in the majority of the other countries and economies.
- Students' exposure to pure mathematics tasks and concepts has a strong relationship with performance in PISA; and the association is stronger for more challenging PISA tasks. In contrast, exposure to simple applied mathematics problems has a weaker association with student performance.
- Around 19% of the performance difference between socio-economically advantaged and disadvantaged students can be attributed to disadvantaged students' relative lack of familiarity with mathematics concepts, on average across OECD countries. Disadvantaged students perform relatively worse on those tasks that require a mastery of symbolic and technical operations and on tasks that test their ability to build mathematic models of reality.

What these results mean for policy

- Students need to be exposed to mathematics content for a sufficient amount of time, but what matters the most is using instruction time effectively.
- Greater exposure to formal mathematics content improves performance – up to a point. All students should be exposed to a curriculum that is coherent across topics and over time, and focuses on key mathematics ideas, so that students can build solid foundations in mathematics.
- Greater familiarity with mathematics may not be sufficient for solving the most complex mathematics problems. Students also need to be exposed to problems that stimulate their reasoning abilities and promote conceptual understanding, creativity and problem-solving skills.
- Disadvantaged students would benefit most from any policy that increases their opportunities to develop not only procedural mathematics skills, but also skills in mathematical modelling.

MATHEMATICS CURRICULA AND PERFORMANCE ON DIFFERENT CONTENT AREAS OF PISA

Not only does PISA assess students on their performance in mathematics, reading and science, but it can also describe students' performance on four distinct mathematical content areas, the "big ideas" that nourish the growing branches of mathematics (Steen, 1990; OECD, 2013a):

- ***Change and relationships***: Tasks related to *change and relationships* require students to use suitable mathematical models to describe and predict change. They often require the application of algebra.
- ***Space and shape***: Tasks related to *space and shape* entail understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. *Space and shape* is the "big idea" most closely related to geometry.
- ***Quantity***: Tasks related to *quantity* involve applying knowledge of numbers and number operations in a wide variety of settings.
- ***Uncertainty and data***: Tasks related to *uncertainty and data* involve knowledge of variation in processes, uncertainty and error in measurement, and chance. This area has a strong connection to probability and statistics.

The PISA test items are split almost evenly across the four content areas (OECD, 2013a).

The four content areas are related to broad parts of the mathematics curriculum found in all PISA countries and economies. Students typically do better on items in which underlying concepts, formats and contexts are familiar to them, than on items in which these aspects are not as familiar. As such, the relative performance of countries across the four content areas (the PISA mathematics content subscales) reflects differences in course content available to 15-year-old students, curriculum priorities, and item difficulty (OECD, 2014). Among other factors, weaker relative performance in a content area might signal an imbalance in the curriculum, which could lead to curriculum reform (Cosgrove et al., 2004).

Shanghai-China significantly outperforms all other countries/economies on each of the mathematics content-area subscales. In relative terms, Shanghai-China performs extraordinarily well on the *space and shape* subscale and less markedly so on the *change and relationships* subscale (Figure 3.1). Several other Asian countries are relatively strong in those tasks that require students to apply geometry.

There are greater international differences in contents and emphasis with geometry curricula than with arithmetic and algebra (French, 2004). Curriculum descriptions collected for the 2011 Trends in International Mathematics and Science Study (TIMSS) show that several Asian economies expose students to advanced spatial mathematics at early grades. For example, in Hong Kong-China, the relationship between three-dimensional shapes and their two-dimensional representations are introduced to students in grade 7, when they are around 13 years old. In Chinese Taipei, students practice the "translation, reflection and rotation" of figures when they are taught the topic of "quadratic function" in grade 9, when they are around 15 years old[1].

At the other end of the spectrum, Ireland performs relatively worse on PISA items in the *space and shape* content area (Figure 3.1). This relative weakness among Irish students might reflect differences between the PISA content area *space and shape*, which focuses more on visualisation skills, and the Irish Junior Certificate Geometry, which emphasises traditional Euclidian geometry (Shiel, 2007).

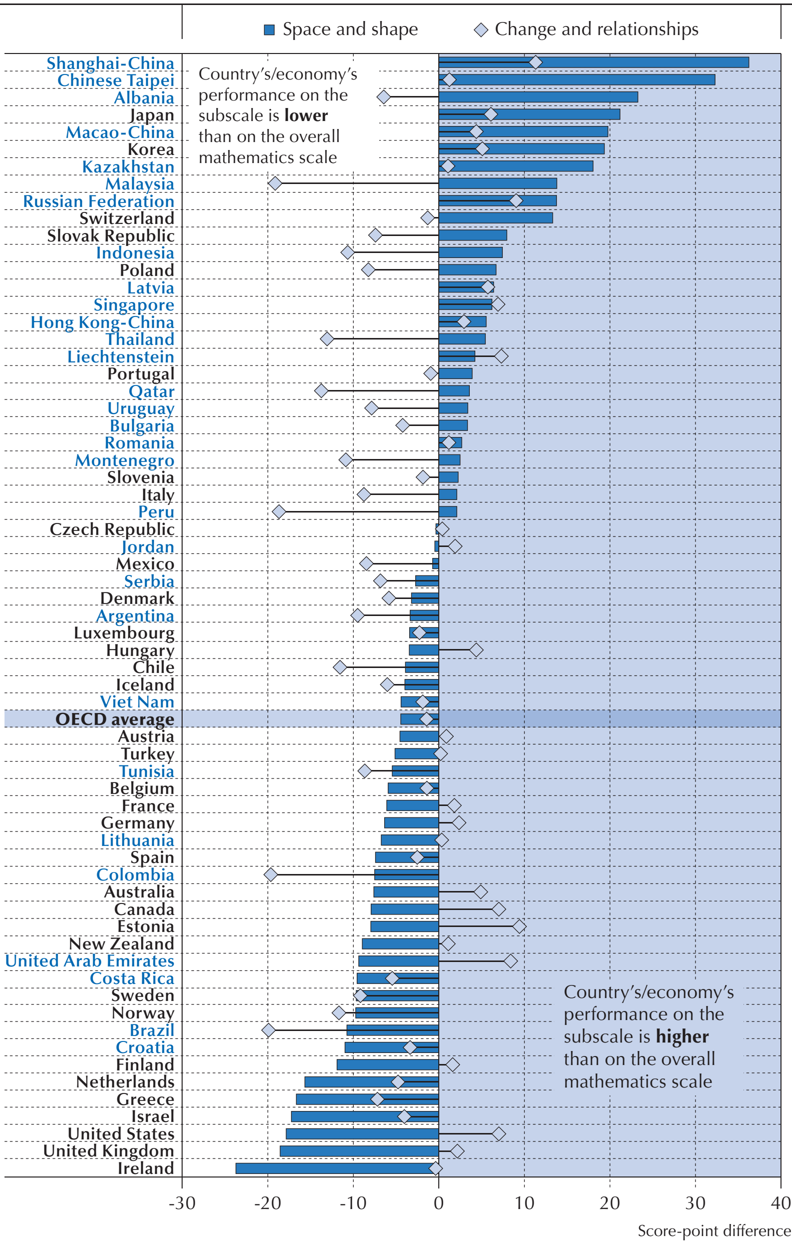
Differences in countries' relative performance should not be solely attributed to variations in how curricula are organised across the different areas of algebra, geometry, quantity and statistics; they may also reflect other characteristics of the individual tasks, such as the task's level of difficulty. An item's difficulty can be described by the percentage of students who responded correctly to it. The analysis in the rest of the chapter and of the report will refer to a logarithmic transformation of this percentage (a logit), where positive logits mean that more than half respondents answered correctly and negative logits mean that fewer than half respondents answered correctly.[2]

Figure 3.2 shows that the items classified in the content areas *space and shape* and *change and relationships* are, on average, much more difficult than the items in the areas of *quantity* and *uncertainty and data*.[3] The item REVOLVING DOOR Question 2 (see the full text of the question at the end of this chapter) is a *space and shape* item and is over 520 points more difficult on the PISA scale than CHARTS Question 1 (see the end of this chapter), an *uncertainty and data* item. The relatively higher performance among Asian countries and economies on tasks requiring greater knowledge of geometry and algebra can thus be explained by Asian students' greater capacities to solve more challenging problems, such as REVOLVING DOOR.

■ Figure 3.1 ■

Performance on the different mathematics content subscales

Score-point difference between the overall mathematics scale and each content subscale



Countries and economies are ranked in descending order of the difference between the overall mathematics score and the score on the mathematics subscale space and shape.

Source: OECD, PISA 2012 Database, Table 3.1.

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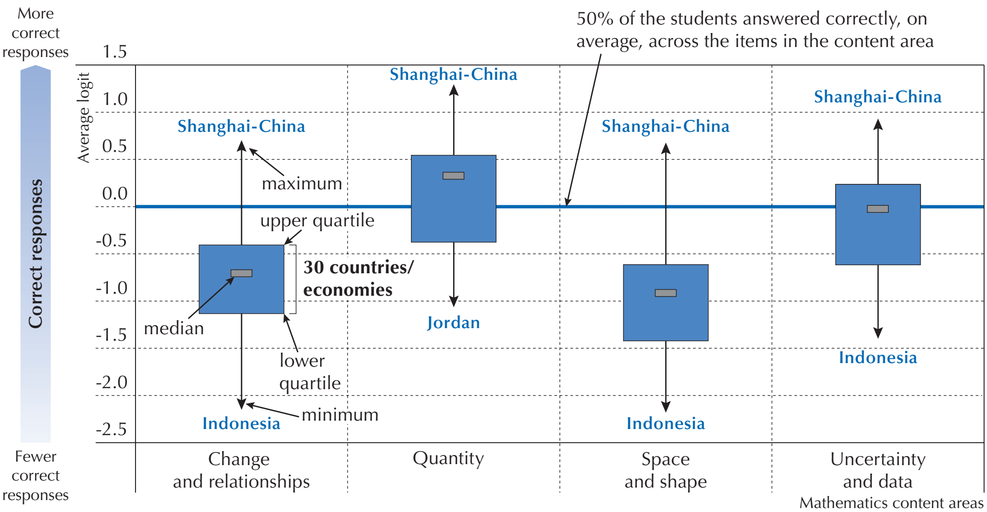
■ Figure 3.2 ■
Difficulty of PISA tasks, by content area
Variation across all countries and economies



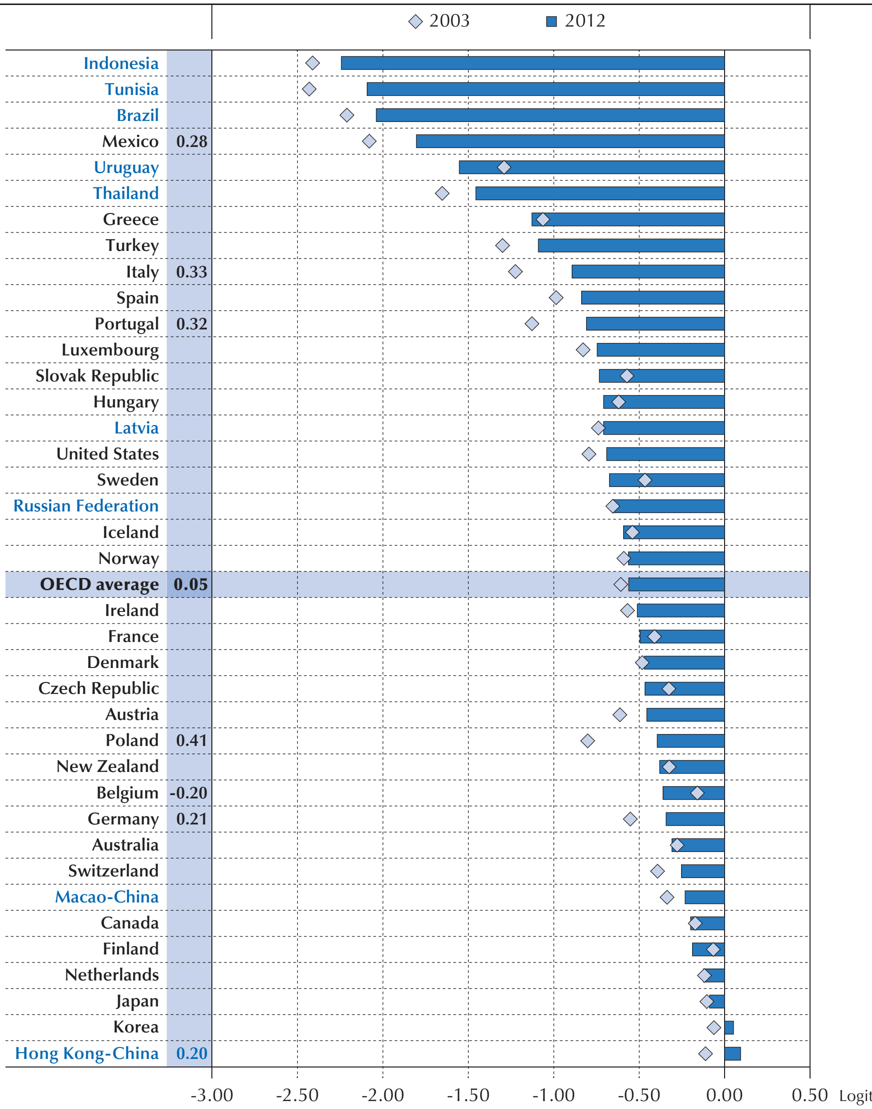
How to read the chart: The figure is a box-and-whisker plot showing the distribution of average logit values in 62 participating countries and economies with available data. For example, in the content area *change and relationships*, Indonesia has the minimum logit value (-2.20), while Shanghai-China has the maximum logit value (0.72) across all countries and economies. A quarter of the countries and economies have logit values included betwen the minimum and the lower limit of the box (-1.13) and a quarter of countries and economies have logit values above the upper limit (-0.41). Half of the countries and economies have logit values included between the lower and upper limit of the box (between -1.13 and -0.41); the horizontal bar represents the median value across all countries and economies (-0.71).
Note: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
Source: OECD, PISA 2012 Database, Table 3.2b.
StatLink *http://dx.doi.org/10.1787/888933377254*

Looking at how the performance of countries has evolved across different content areas of mathematics reveals interesting trends, possibly related to changes in the focus of mathematics instruction. Figures 3.3a-d show trends in OECD countries' performance on the 31 mathematics items that were used in both the 2003 the 2012 assessments. On average across OECD countries, the percentage of students who were able to answer correctly the questions related to *change and relationships* increased, remained virtually the same for the questions related to *uncertainty and data*, but decreased for questions related to *quantity* and, more significantly, *space and shape*.

Performance on items related to *change and relationships* improved substantially in Italy, Poland and Portugal (over 0.4 logits in Poland, corresponding approximately to an increase of 7 points in the percentage of students answering correctly to those items, see Tables 3.3a and 3.3b). The Czech Republic, France, the Slovak Republic, Sweden and Uruguay saw a large deterioration in their students' performance on items related to *space and shape* that were tested in both 2003 and 2012 (0.5 logits in Uruguay, corresponding approximately to a decrease of 7 points

■ Figure 3.3a ■

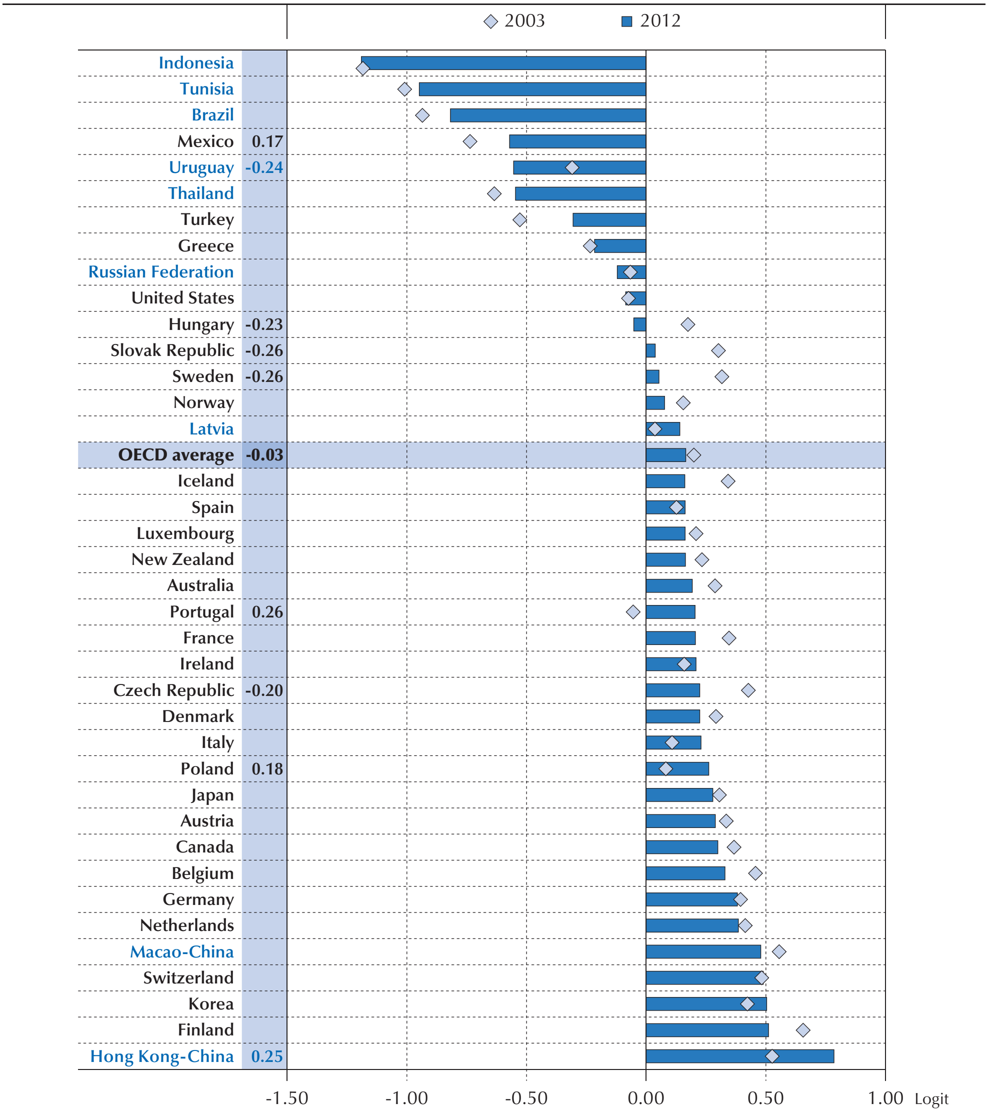
Change between 2003 and 2012 in mathematics performance across content area *change and relationships*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty. The logit coefficients are calculated over the 7 items in *change and relationships* assessed both in 2003 and in 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area change and relationships *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink http://dx.doi.org/10.1787/888933377262

■ Figure 3.3b ■

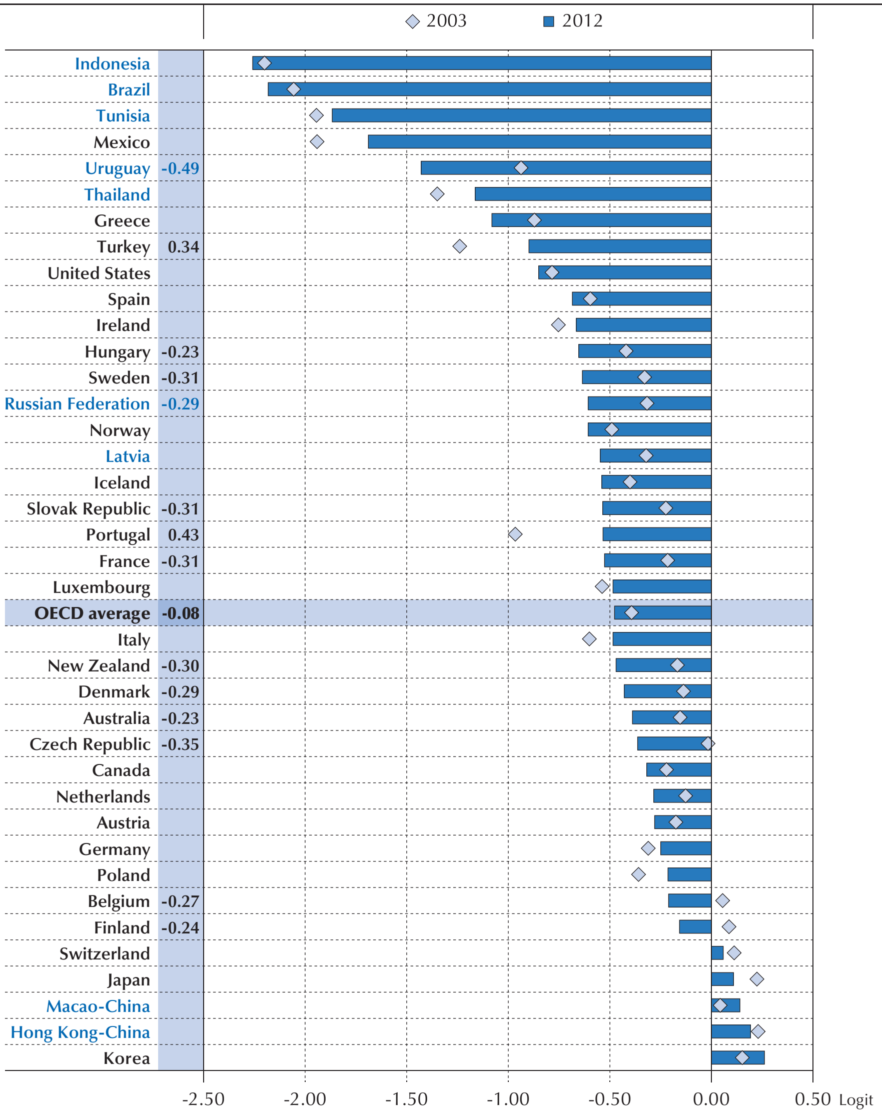
Change between 2003 and 2012 in mathematics performance across content area *quantity*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
The logit coefficients are calculated over the 8 items in *quantity* assessed both in 2003 and 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area quantity *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink *http://dx.doi.org/10.1787/888933377278*

■ Figure 3.3c ■

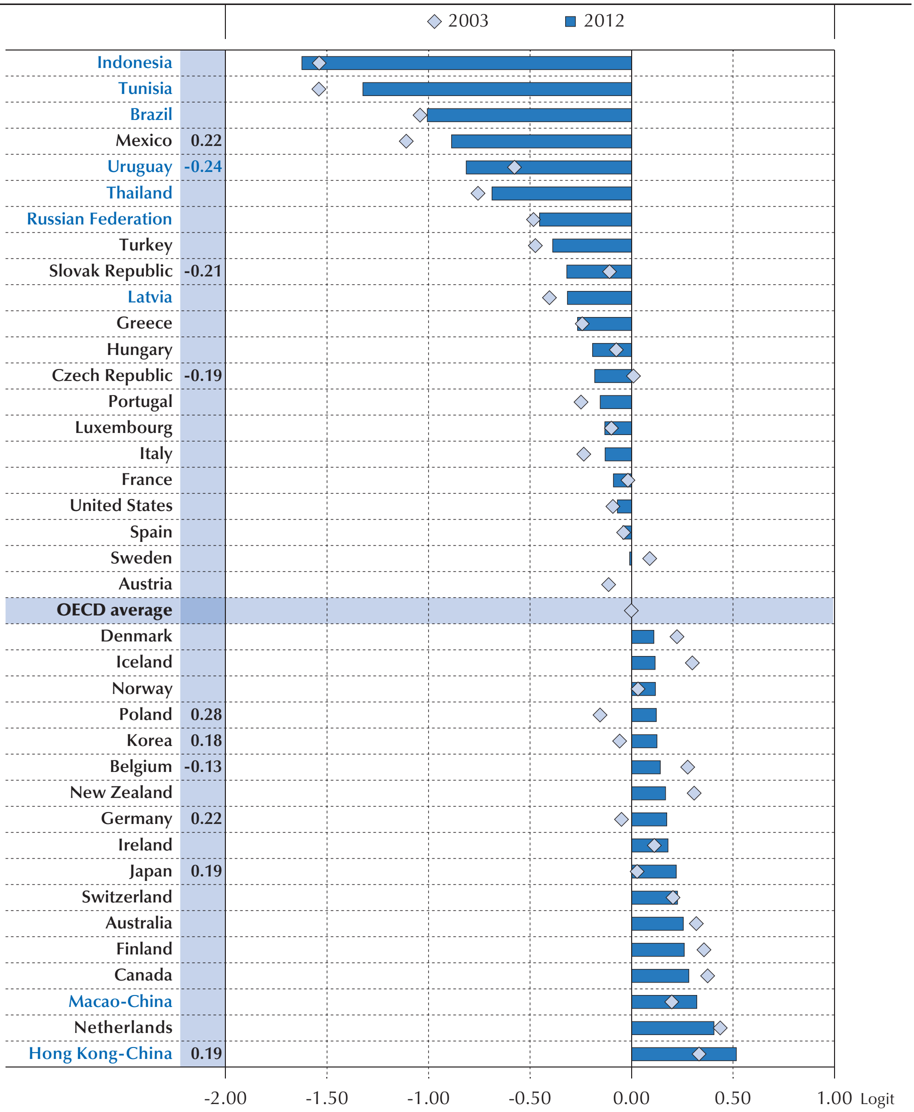
Change between 2003 and 2012 in mathematics performance across content area *space and shape*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty. The logit coefficients are calculated over the 9 items in *space and shape* assessed both in 2003 and in 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area space and shape *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink http://dx.doi.org/10.1787/888933377284

■ Figure 3.3d ■

Change between 2003 and 2012 in mathematics performance across content area *uncertainty and data*



Notes: A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
The logit coefficients are calculated over the 7 items in *uncertainty and data* assessed both in 2003 and in 2012.
Only countries and economies with comparable data from PISA 2003 and PISA 2012 are shown.
The OECD average is calculated as the average of 29 countries.
Only statistically significant average logit differences between 2012 and 2003 are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the average logit of questions in the content area uncertainity and data *in 2012.*
Source: OECD, PISA 2012 Database, Table 3.3a.
StatLink http://dx.doi.org/10.1787/888933377292

in the percentage of students answering those items correctly, see Tables 3.3a and 3.3b). The deterioration in performance on tasks in the *space and shape* area is possibly a consequence of a de-emphasis on geometry in mathematics curricula in some countries (Lehrer and Chazan, 1998). This negative trend deserves further consideration, as geometry is often the only visually oriented mathematics that students are offered. Students who receive training in the analysis of shapes develop their abilities to "see" the end product in their mind's eye, inspect its individual elements, and make sufficiently good conjectures about their relationships. All of these skills are essential, not only for scientists but for everyone in their daily lives.

VARIATIONS IN OPPORTUNITY TO LEARN AND PERFORMANCE IN MATHEMATICS

Variations in time spent learning

The amount of time that students spend learning is a basic component of their opportunity to learn (Carroll, 1963; Gromada and Shewbridge, 2016). PISA 2012 data based on students' reports show that, on average across OECD countries, students spend about 3 hours and 38 minutes per week in mathematics class, 3 hours and 35 minutes in language-of-instruction class, and 3 hours and 20 minutes in science class, though the time varies considerably within countries (OECD, 2013b, Table IV.3.21).

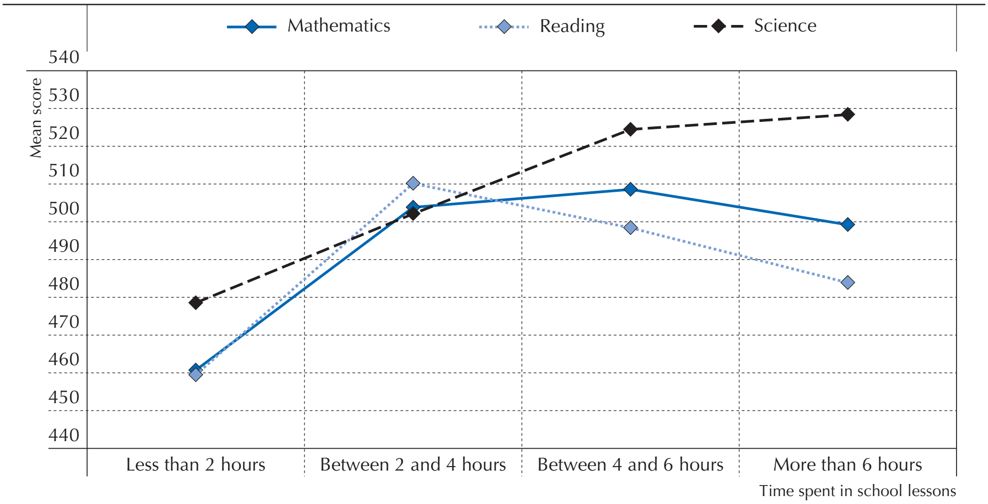
On average across OECD countries, learning time in regular mathematics lessons is positively correlated to student performance, even after accounting for various student and school characteristics, including socio-economic status (OECD, 2013b, Table IV.1.12c). However, Figure 3.4 shows that this relationship is not linear. An increase in class time of up to four hours per week is associated with a large improvement in performance in the three PISA subjects. After that threshold, longer instruction time is associated with smaller improvements in science performance and a deterioration in reading performance. More than six hours per week of class time is also associated with a slight deterioration in mathematics performance. PISA 2006 data also show that performance in mathematics and reading starts deteriorating moderately after six or more hours of instruction per week (OECD, 2011). One possible explanation for the difference across the three subjects is that the students who spend a long time in regular school science lessons choose to do so in enrichment courses, because they are interested in science and attend schools with the resources and facilities to offer such courses, while students who spend a long time in regular school mathematics or language-of-instruction lessons are obliged to do so for remedial purposes.

Besides remedial education, other school- and student-level differences can affect the time-achievement relationship. For instance, better performing students may be more likely to be enrolled in academic school tracks, to receive higher-quality instruction, and to be sorted into better classroom or school environments where they also receive longer instruction time, thus making it difficult to say whether longer time increases performance or whether better performing students receive longer instruction for other reasons. A number of studies have attempted to pin down the causal effect of instruction time on achievement by accounting for characteristics of students and schools (Lavy, 2015; Rivkin and Schiman, 2015) or by looking for variations in learning time not attributable to students', parents' or schools' behaviour, such as those related to school reforms or to unscheduled school closing due to snow (Bellei, 2009; Lavy, 2012;

■ Figure 3.4 ■

Relationship between performance and time spent in school lessons

OECD average



Note: The OECD average performance in each subject is calculated only for countries with a valid score across all four time brackets.
Source: OECD, PISA 2012 Database, Table 3.4a.
StatLink *http://dx.doi.org/10.1787/888933377309*

Marcotte, 2007; Marcotte and Hemelt, 2008; Pischke, 2007). They have generally found a positive relationship between instruction time and performance.

Following an approach similar to that of Rivkin and Schiman (2015), the analysis reflected in Figure 3.5 tries to isolate the link between instruction time and performance by showing how the score-point difference between mathematics and reading performance changes when the difference in instruction time for the two subjects increases by one hour.[4] These estimates are based on differences in time and performance between subjects among students enrolled in the same school and grade, so as to reduce possible influences coming from the fact that better performing students get sorted into schools and grades providing longer instruction time in mathematics.

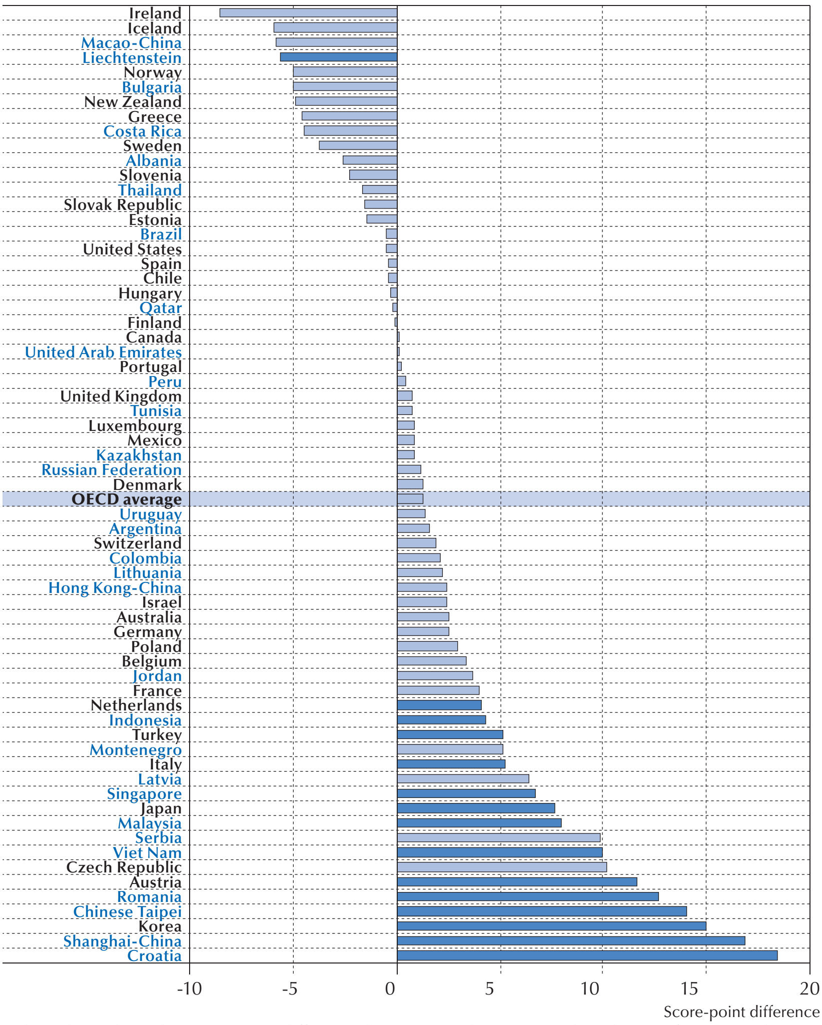
Figure 3.5 shows that mathematics performance improves compared with reading performance when the difference in instruction hours between mathematics and reading increases by one hour (that is, when students receive one hour more of instruction in mathematics per week than instruction in reading). This is observed in Austria (an improvement of 12 score points), Croatia (18 points), Indonesia (4 points), Italy (5 points), Japan (8 points), Korea (15 points), Malaysia (8 points), the Netherlands (4 points), Romania (13 points), Shanghai-China (17 points), Singapore (7 points), Chinese Taipei (14 points), Turkey (5 points) and Viet Nam (10 points).

Overall, this analysis of the relationship between instruction time and performance suggests that re-allocating time across subjects (moderately) increases students' performance in PISA only in a minority of countries and economies but that it does not automatically affect performance

■ Figure 3.5 ■

Mathematics performance and instruction time, after accounting for school characteristics

Score-point difference between mathematics and reading performance associated with a one-hour difference between mathematics and reading instruction time



Notes: The chart shows how the score-point difference between mathematics and reading performance changes when the difference in the amount of time devoted to mathematics with respect to reading instruction increases by one hour. In the Netherlands, for example, student performance in mathematics improves by four points compared with reading performance if the difference between the hours of mathematics and the hours of reading classes increases by one hour. The differences in performance and instruction time are calculated as averages for students in the same school and grade, and account for the observable and unobservable characteristics of schools that might influence the relationship between hours of instruction and student performance.
Statistically significant score-point differences are marked in a darker tone.
Countries and economies are ranked in ascending order of the effect of an additional hour of mathematics instruction on performance in mathematics compared with reading performance.
Source: OECD, PISA 2012 Database, Table 3.5.
StatLink http://dx.doi.org/10.1787/888933377313

on a large scale. In other words, the relationship between instruction time and performance turns out to be relatively weak after taking into consideration that better schools provide more instruction time. These results are not surprising considering that the PISA questionnaire provides information on the instruction time allocated to subjects, but not on the time that students spend engaged in learning.

If the quality of time spent learning in the classroom is poor, longer instruction time will not translate into greater opportunity to learn (Gromada and Shewbridge, 2016). More positive classroom environments – better student behaviour and good teacher-student relations – appear to augment the benefit of additional instruction time (Rivkin and Schiman, 2015). Figure 3.6 shows the distribution of mathematics performance and of the *index of disciplinary climate* by the time per week students spend in mathematics class.[5] In Korea, students who are exposed to mathematics for a longer time score higher and enjoy a positive learning climate. This suggests that good class discipline allows long instruction hours to be more productive. By contrast, the longer the time students in Switzerland spend in mathematics classes, the more they report poor performance and a poor classroom climate, suggesting that low-performing students spend more hours in the classroom, but these extra hours may fail to improve their performance because time is lost in noise and disorder.

In addition to regular classes, students are increasingly offered the opportunity to attend programmes providing additional instruction in school subjects outside of regular classes (Kidron and Lindsay, 2014). In PISA 2012, students reported information about the time they spend in after-school lessons offered at their school, at their home or somewhere else. On average across OECD countries, 38% of students reported that they attend after-school lessons in mathematics, 27% attend after-school lessons in the language of instruction, and 26% attend such lessons in science (OECD, 2013b: Table IV.3.25).

For all subjects, there is a negative correlation between performance and the time spent studying after school, on average across OECD countries (Figure 3.7). Again, this relationship should not be interpreted as causal, as low-performing students are more likely to participate in after-school remedial courses or personal tutoring. Japan and Korea are exceptions, as students in these countries who spend more hours in after-school mathematics lessons are also high performers in mathematics (Table 3.4b). In these two countries, after-school supplementary courses are often intended to help students master academic subjects, improve their performance, and ultimately earn good scores on high-stake tests, such as the competitive college entrance examination (Park, 2013).

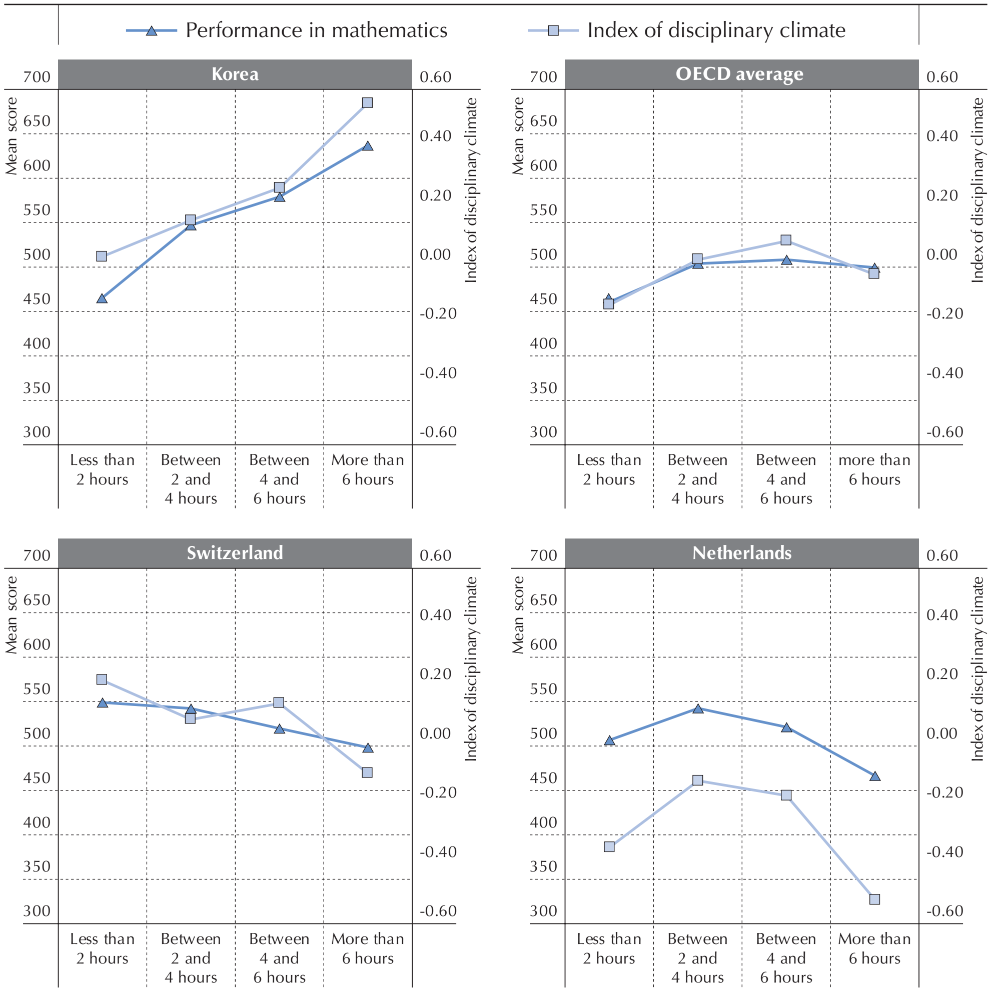
Variations in exposure to and familiarity with mathematics

Opportunity to learn refers not only to the time a student spends learning given content, but also, and more importantly, to the content taught in the classroom. International and country-level research has highlighted a positive association between content coverage and achievement in mathematics (Dumay and Dupriez, 2007; Rowan, Correnti, and Miller, 2002; Schmidt et al., 2001; Schmidt et al., 2011) and science (Sousa and Armor, 2010). This section extends previous analyses of PISA 2012 data investigating the link between exposure to and familiarity with mathematics on the one hand and mathematics performance on the other (OECD, 2014; Schmidt, Zoido and Cogan, 2014; Schmidt et al., 2015).

■ Figure 3.6 ■

Time spent in mathematics lessons, performance in mathematics and disciplinary climate

In hours per week



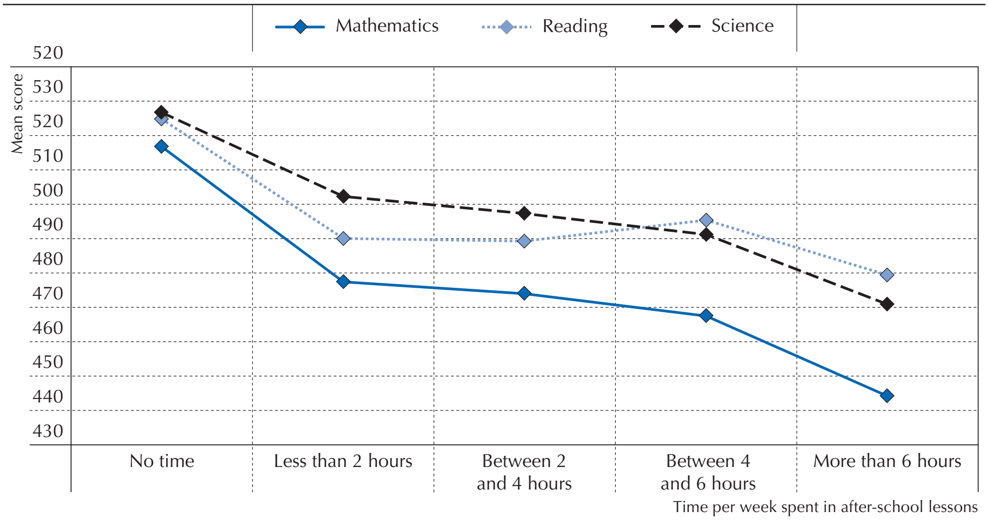
Notes: The *index of disciplinary climate* summarises students' reports on the frequency of noise, disorder and inactivity in the classroom due to disciplinary issues.
The OECD average of the *index of disciplinary climate* and of mathematics performance is calculated only for countries with a valid score across all four time brackets.
Source: OECD, PISA 2012 Database, Tables 3.4a and 3.6.
StatLink *http://dx.doi.org/10.1787/888933377320*

PISA data show that students' mathematics performance is positively associated with their exposure to pure and applied mathematics as well as with their familiarity with mathematics concepts.

■ Figure 3.7 ■

Relationship between performance and time spent in after-school lessons

In hours per week, OECD average



Notes: After-school lessons include lessons in subjects that students are also learning at school, on which they spend extra learning time outside of normal school hours. The lessons may be given at their school, at home or somewhere else.
The OECD average performance in each subject is calculated only for countries with a valid score across all the five time brackets.
Source: OECD, PISA 2012 Database, Table 3.4b.
StatLink *http://dx.doi.org/10.1787/888933377330*

First, Figure 3.8a shows that more frequent exposure to pure mathematics concepts is associated with better mathematics performance. On average across OECD countries, a one-unit increase in the *index of exposure to pure mathematics* is associated with an increase of 30 score points in mathematics performance. The link between exposure to pure mathematics and achievement is particularly strong in Korea, the Netherlands, New Zealand, Singapore and Chinese Taipei, where a one-unit increase in exposure to pure mathematics is related to an increase of more than 40 score points in mathematics performance.

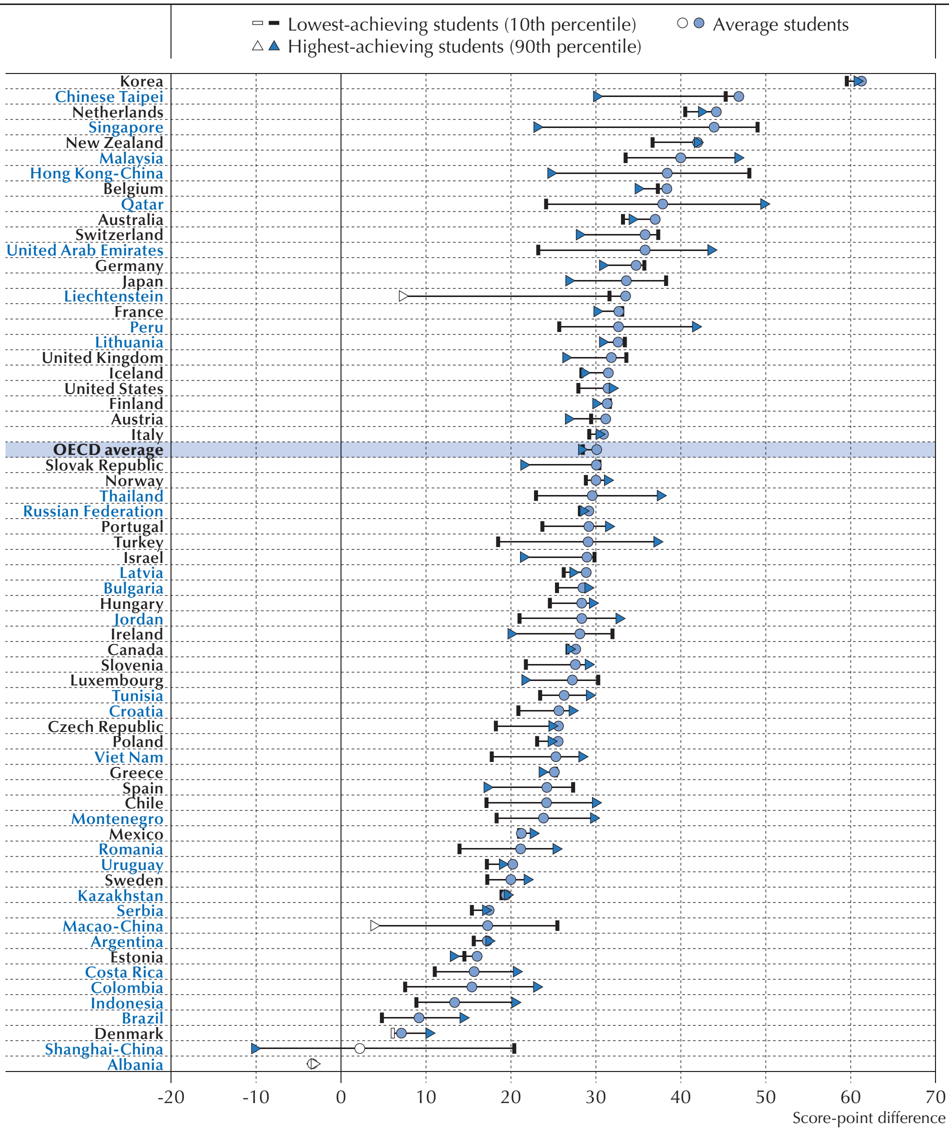
Second, more frequent exposure to applied mathematics is also related to mathematics performance in most countries (Figure 3.8b), even though the effect is weaker than that linked to exposure to pure mathematics. On average across OECD countries, a one-unit increase in the *index of exposure to applied mathematics* is associated with an increase of about 9 score points in mathematics performance. The effect is strongest in Australia, Finland, Japan, Korea, New Zealand, Chinese Taipei and the United Kingdom (more than 20 score points) while it is negative in Greece, Shanghai-China, the Slovak Republic, Spain, Turkey and Uruguay.

The association between exposure to applied mathematics and mathematics performance is weaker than that between pure mathematics and mathematics performance (or even negative). This may be due to reverse causality. The mathematics tasks PISA uses to measure exposure to

■ Figure 3.8a ■

Relationship between exposure to pure mathematics and mathematics performance

Score-point difference in mathematics performance associated with a one-unit increase in the index of exposure to pure mathematics

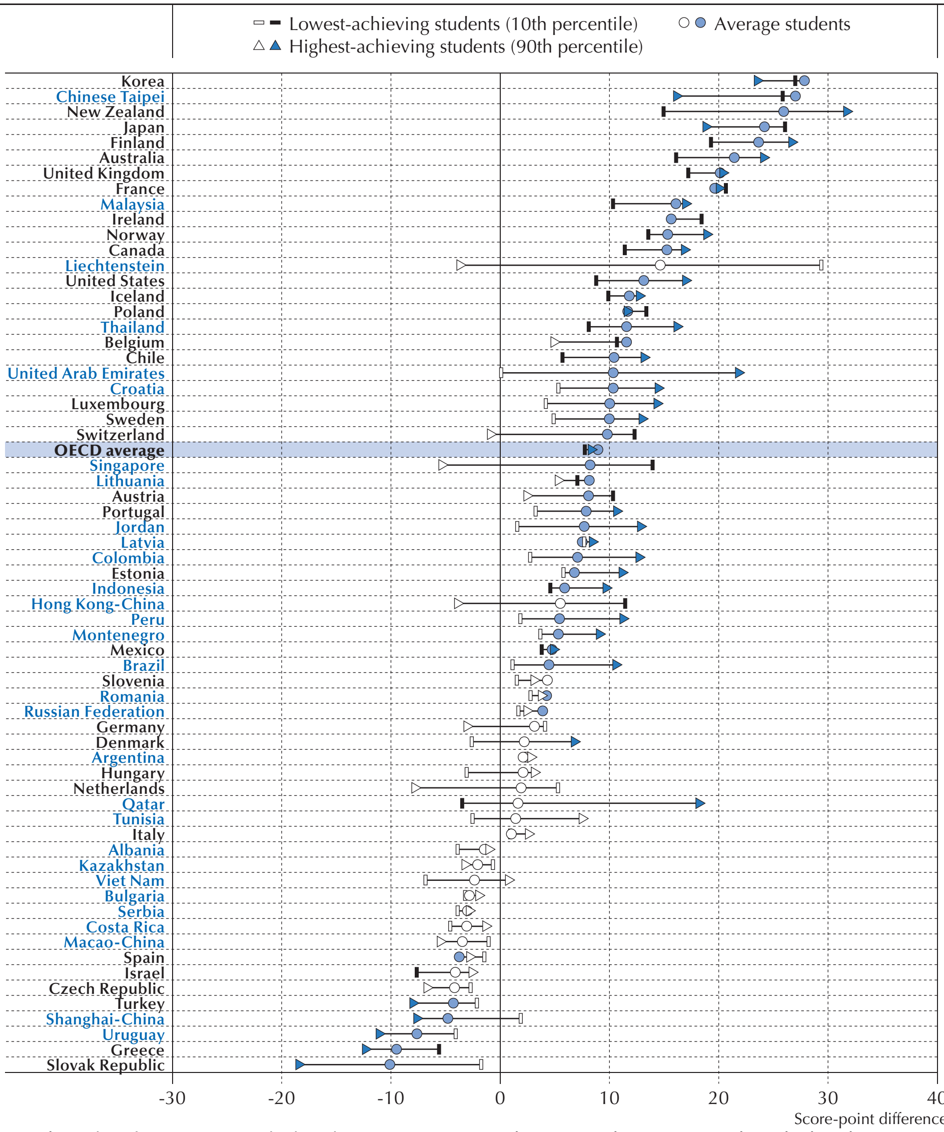


Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in descending order of the score-point difference associated with a one-unit change in the index of exposure to pure mathematics *for the average students.*
Source: OECD, PISA 2012 Database, Table 3.7.
StatLink http://dx.doi.org/10.1787/888933377349

■ Figure 3.8b ■

Relationship between exposure to applied mathematics and mathematics performance

Score-point difference in mathematics performance associated with a one-unit increase in the index of exposure to applied mathematics



Notes: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in descending order of the score-point difference associated with a one-unit change in the index of exposure to applied mathematics *for the average students.*

Source: OECD, PISA 2012 Database, Table 3.7.

StatLink http://dx.doi.org/10.1787/888933377355

applied mathematics are relatively easy for 15-year-old students (e.g. working out from a train timetable how long it would take to get from one place to another), and low-performing students are more likely than higher-achieving students to have been exposed to this type of problem.

Third, mathematics performance is also related to greater familiarity with mathematics concepts (Figure 3.8c), the measure that better captures the cumulative opportunity to learn mathematics content over a student's career. On average across OECD countries, a one-unit increase in the *index of familiarity with mathematics* (equivalent to the difference between having heard of a series of mathematics concepts "often" and "a few times"; see Chapter 1) corresponds to a 41 score-point increase in mathematics performance. This association is stronger (an increase of more than 50 score points) in Australia, Korea, New Zealand and Chinese Taipei.

PISA asked students how frequently they are exposed to specific problems during mathematics lessons or assessments, including algebraic word problems, procedural tasks, contextualised mathematics problems, and pure mathematics problems (see Box 1.2 in Chapter 1 and questions at the end of Chapter 1 for some examples). Tables 3.8a-d show that students who are frequently exposed to these problems in mathematics classes perform better in mathematics than students who are never exposed to them. Across the four types of tasks, the relationship between exposure and performance is strongest for procedural tasks and weakest for contextualised mathematics tasks, on average across OECD countries. This confirms that exposure to pure mathematics is more closely related to performance in PISA than exposure to applied mathematics is. The results suggest that the use of real-life examples is not enough to transform routine problems into challenging mathematics problems that build mathematics literacy. Students – and low-achieving students in particular – might also have problems in transferring what they learn in a specific context to other contexts (see Box 1.3 in Chapter 1). When interpreting these results, it is important to keep in mind that it is much easier to measure exposure to pure mathematics than to measure exposure to applied and contextualised mathematics, given that applied mathematics problems are, by nature, more ambiguous and diverse.

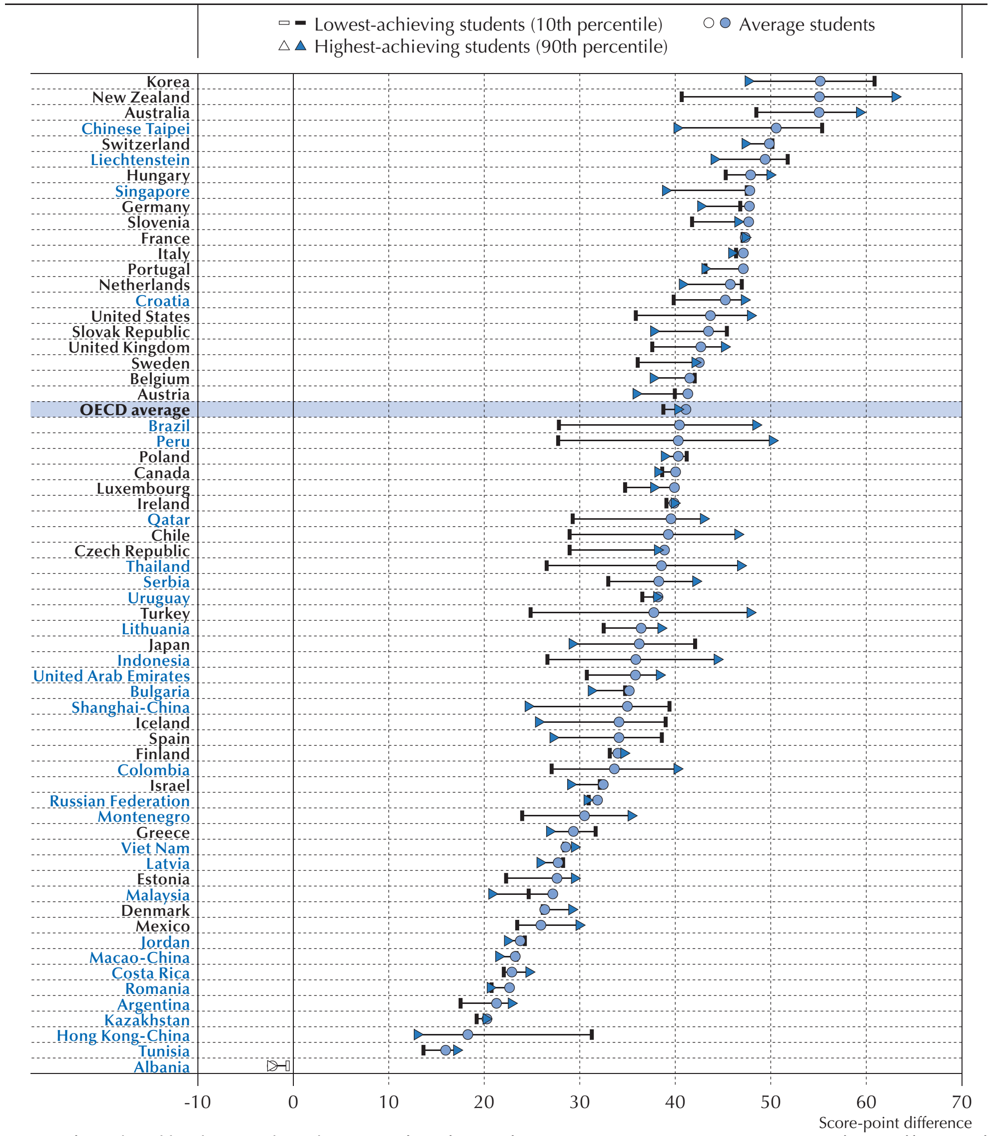
In most countries and economies, the association between opportunity to learn and mathematics performance is stronger among high-achieving students than among low-achieving students (Figures 3.8a, 3.8b and 3.8c). In Brazil, New Zealand, Peru, Thailand and Turkey, the effect of familiarity with mathematics on performance is more than 20 score points larger for the 10% of students with the highest scores than for the 10% of students with the lowest scores. In these countries, it is possible that high-achieving students can better profit from what they are taught but it may also be that they are exposed to a more advanced curriculum (Table 3.7).

But in a number of countries and economies, the reverse is true: the association between exposure to/familiarity with mathematics and mathematics performance is stronger among low achievers than among high achievers. For instance, in Hong Kong-China, Liechtenstein, Macao-China, Shanghai-China, Singapore and Chinese Taipei, the difference in performance related to frequent exposure to pure mathematics is at least 15 score points larger among low achievers than among high achievers (Figure 3.8a and Table 3.7). Perhaps, in these countries and economies, mathematics concepts are taught in such an accessible way that low-performing students benefit even more than high-performing students do, thus suggesting that the organisation of curriculum and teaching

■ Figure 3.8c ■

Relationship between familiarity with mathematics and mathematics performance

Score-point difference in mathematics performance associated with a one-unit increase in the index of familiarity with mathematics



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential functions, divisor, quadratic function, etc.).
The OECD average for familiarity with mathematics concepts is thus calculated as the average of 33 countries.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in descending order of the score-point difference associated with a one-unit change in the index of familiarity with mathematics *for the average students.*
Source: OECD, PISA 2012 Database, Table 3.7.
StatLink *http://dx.doi.org/10.1787/888933377366*

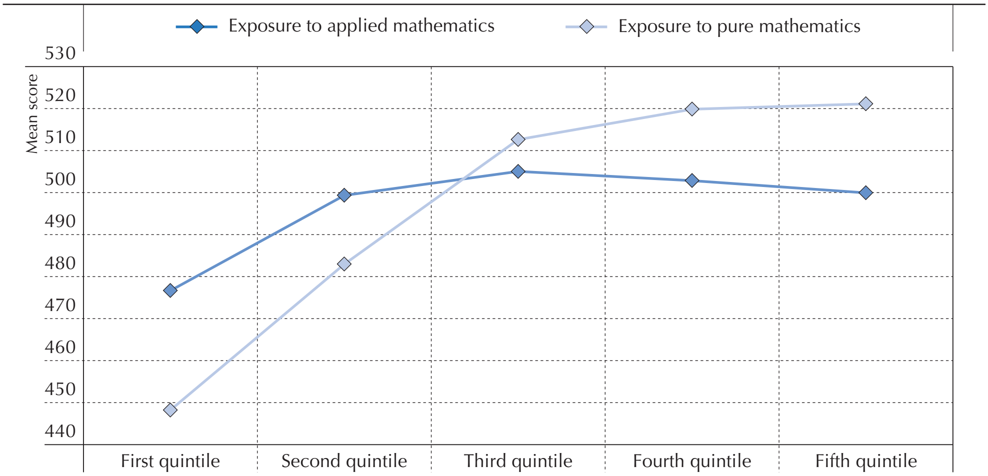
can help to narrow the achievement gap. Chapter 5 will discuss these policies and provide more examples.

The link between exposure to mathematics content and performance varies by the frequency of exposure. Figure 3.9 shows that more frequent exposure to pure mathematics is associated with smaller improvements in performance than less frequent exposure is. Figure 3.9 also shows that mathematics performance improves slightly between the first and the third quintiles of exposure to applied mathematics and decreases slightly among students who reported more frequent exposure (the fourth and fifth quintiles). Again, the slightly negative association between performance and a very frequent exposure to applied mathematics is unlikely to mean that greater exposure to applied mathematics reduces performance; it may rather come from the fact that the mathematics tasks PISA uses to measure exposure to applied mathematics are relatively easy and may be used to make mathematics accessible to low-performing students.

■ Figure 3.9 ■

Performance in mathematics, by exposure to applied and pure mathematics

OECD average



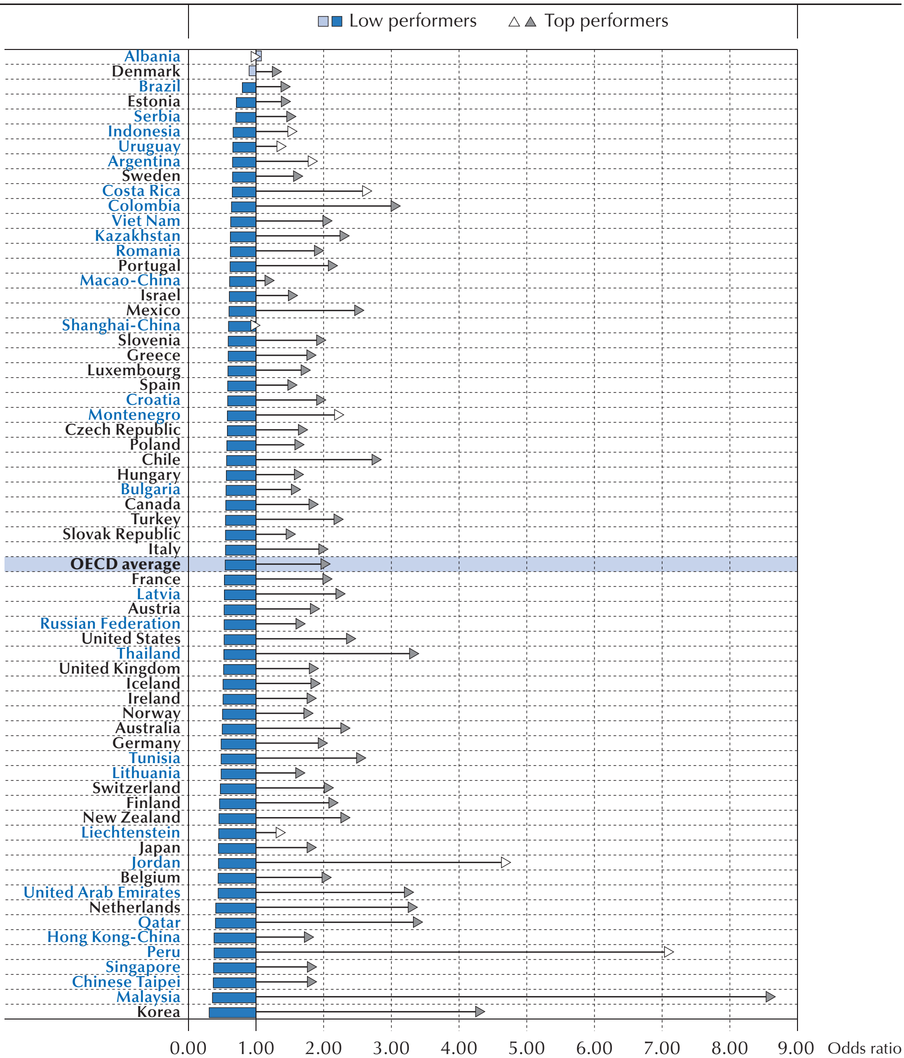
Notes: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Source: OECD, PISA 2012 Database, Table 3.9.
StatLink *http://dx.doi.org/10.1787/888933377377*

Not only is exposure to mathematics associated with average performance, it is also related to a student's chances of being a low performer (that is, of performing at or below Level 1 on the PISA mathematics scale) or a top performer (that is, of performing at or above Level 5). Figure 3.10 shows that, for an average student in an OECD country, a one-unit increase in the *index of exposure to pure mathematics* doubles the likelihood of being a top performer,

■ Figure 3.10 ■

Exposure to pure mathematics and the likelihood of top and low performance

Change in the likelihood of low and top performance associated with a one-unit change in the index of exposure to pure mathematics



How to read the chart: An *odds ratio* of two for top-performance means that a one-unit increase in the *index of exposure to pure mathematics* doubles the probability that the student is a top performer in mathematics. Similarly, an odds ratio of 0.5 for low performance means that a one-unit increase in the *index of exposure to pure mathematics* reduces the probability of low performance by half.

Notes: Low performers are students who score below proficiency Level 2. Top performers are students who score at or above proficiency Level 5.

Values that are statistically significant are marked in a darker tone.

Countries and economies are ranked in descending order of the effect of exposure to pure mathematics on the likelihood of low performance.

Source: OECD, PISA 2012 Database, Table 3.10.

StatLink http://dx.doi.org/10.1787/888933377389

and reduces by half the probability that this student is a low performer. In Colombia, Korea, Malaysia, the Netherlands, Qatar, Thailand and the United Arab Emirates, a one-unit increase in the *index of exposure to pure mathematics* triples the likelihood that a student is a top performer.

The positive association between exposure to mathematics and performance in PISA does not necessarily indicate a causal link, as the direction of causality is not clear. Greater exposure to mathematics may improve performance, but at the same time, students with greater mathematical ability and motivation may choose – or be sorted into – schools that offer them greater exposure to mathematics.

This causality problem can be mitigated by analysing how different levels of exposure to mathematics influence the performance of students who attend the same school, as this excludes school factors that might influence the relationship. PISA randomly selects students of a given age within each school, so for the majority of PISA countries and economies it is possible to compare students who attend different grades within the same school. As shown in Chapter 1 (Figure 1.9), students in higher grades are more frequently exposed to pure mathematics. To investigate the relationship between exposure to pure mathematics and performance, the analysis will look at differences in exposure between students who attend different grades within the same school. This method accounts for differences across schools that might have an influence on the results presented so far (such as the fact that better performing students may choose schools that offer them more mathematics instruction) but does not account for ways in which students may be sorted within schools (for example, through ability grouping within the school).

Figure 3.11 shows that, across OECD countries, students who attend a higher grade have higher mathematics performance by 29 score points than students in the same school who attend a lower grade because they are more exposed to pure mathematics (by one index point).[6] Across students in the same school, the improvement in performance associated with greater exposure is larger than 50 score points in Korea, Luxembourg, Malaysia, Qatar and Spain. This implies that offering all students the opportunity to be exposed to a coherent curriculum does matter for mathematics performance.

Familiarity with mathematics and problem-solving skills

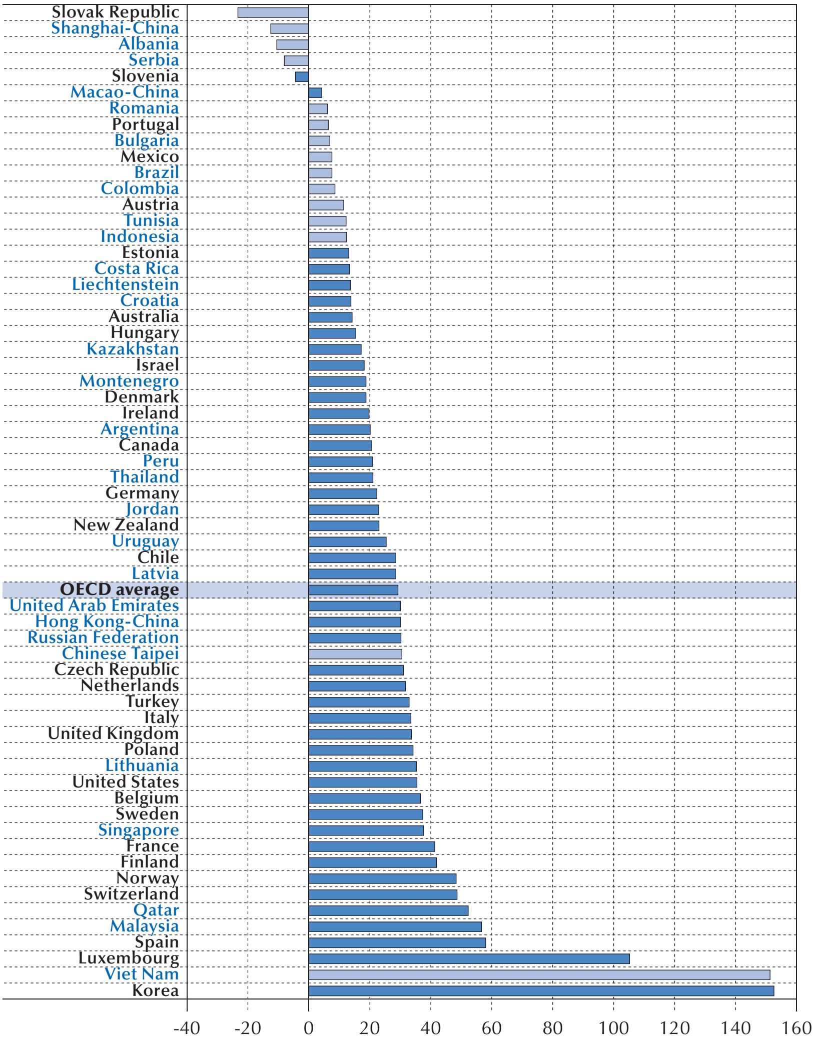
In order to determine how familiarity with mathematics is related to performance on demanding tasks, another analysis focuses on student performance on PISA tasks of various levels of difficulty or requiring different skills (Box 3.1 provides more details on analysis at the task level). Does greater exposure to and greater familiarity with mathematics mean that students will be equipped with all the skills they need to face complex mathematics problems?

Figure 3.12 shows that the greater students' familiarity with mathematics concepts, the more mathematics items (on the paper-based PISA assessment[7]) students answer correctly, on average across OECD countries. The association is stronger for more challenging tasks. For example, a one-unit increase in the *index of familiarity with mathematics* more than doubles (2.6 times) the likelihood that students answer correctly a difficult item like ARCHES Question 2 (difficulty of 785 on the PISA scale), but raises by only 1.5 times the likelihood of a correct response to an easy

■ Figure 3.11 ■

Difference in mathematics performance across grades related to exposure to pure mathematics

Score-point difference between grades in the same school associated with a one-unit difference in the index of exposure to pure mathematics



How to read the chart: On average across OECD countries, 15-year-old students in one grade score 29 points more in mathematics than students one grade below in the same school if the difference in the *index of exposure to pure mathematics* between the two grades is equal to one unit. The estimates can be interpreted as the effect of pure mathematics on perfomance after accounting for observable and unobservable differences across schools.

Notes: Only students in the modal grade and one grade below or above the modal grade are included in the analysis.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in ascending order of the score-point difference between grades.

Source: OECD, PISA 2012 Database, Table 3.11.

StatLink http://dx.doi.org/10.1787/888933377391

Box 3.1. Analysis of performance in PISA at the task level

One of the key strengths of PISA is that the assessment items (tasks) vary considerably in the type of response format, the contexts in which the problem sets are framed, and the type of content knowledge and cognitive processes that they aim to assess. The PISA mathematics tasks can be classified according to what is required of students, including their:

- knowledge of the mathematics **content areas** of *change and relationships*, *space and shape*, *quantity* and *uncertainty and data*. These overarching "big ideas" guide the conceptual understanding of traditional mathematics topics, such as algebra and functions, geometry and measurement.
- capacity to address problems framed in **contexts** dealing with personal life, the social environment in which students live, the world of work, or the use of mathematics in science and technology.
- capability to engage in the cognitive **processes** needed to cover the full cycle of mathematics modelling – formulate, employ and interpret.

PISA items are ranked according to their level of difficulty. The difficulty of the items is calculated after the test is conducted, using a scaling approach known as Item Response Theory, which estimates the difficulty of items and students' score on the test simultaneously. The lower the percentage of students who give the correct answer, the more difficult the item. For example, students with a score of 348 points have a 62% probability (this probability was chosen by the PISA consortium as part of the scoring design) of solving CHARTS Question 1 (see examples at the end of the chapter).[8] This item thus sits at 348 points on the PISA scale of difficulty. Analysing performance at the task level allows for identifying the relative strengths and weaknesses of countries or groups of students in particular areas and processes of mathematics. This section looks at how students perform across items at different levels of difficulty and focuses on four PISA tasks:

Description	Content	Process	Percentage of correct responses at the international level
CHARTS Question 1			
The students are shown a bar graph reporting the sales of CDs from four music bands. They need to identify and extract a data value from the chart to answer a simple question.	Uncertainty and data	Formulate	87%
DRIP RATE Question 1			
The task's stimulus explains a formula used by nurses to calculate the drip rate (in drops per minute) for infusions. The students need to interpret an equation linking four variables, and provide an explanation of the effect of a specified change to one variable on a second variable if all the other variables remain unchanged.	Change and relationships	Employ	22%

...

Description	Content	Process	Percentage of correct responses at the international level
REVOLVING DOOR Question 2			
The task describes a revolving door and presents diagrams that include information above the diameter and different positions of the door wings. The students are asked to calculate the maximum arc length that each door can have so that air never flows between entrance and exit. Students not only have to apply their knowledge of circle geometry (the formula of the circumference) but also engage in sophisticated reasoning to formulate a mathematical model. The task gives students no suggested approaches so that they have to invent their own strategies.	Space and shape	Formulate	3%
ARCHES Question 2			
The problem contains technical terms that students need to interpret in relation to a diagram. The students are asked to formulate a geometric model and apply their procedural knowledge of trigonometry or of the Pythagorean theorem to calculate a length.	Space and shape	Formulate	5%

item like CHARTS Question 1 (difficulty of 348). Students who solve ARCHES can interpret a text containing technical terms, and apply their procedural knowledge (trigonometry or Pythagorean theory) to calculate a length.

Knowing mathematics terminology, facts and procedures has a positive impact on overall performance and is even more valuable for solving more challenging problems. It may seem that if you want to improve students' ability to solve difficult mathematics problems, you may just extend the coverage of the mathematics curriculum and give students more time to practice their procedural skills. But that is only partly true. It takes more than content knowledge and practice to develop a good problem solver.

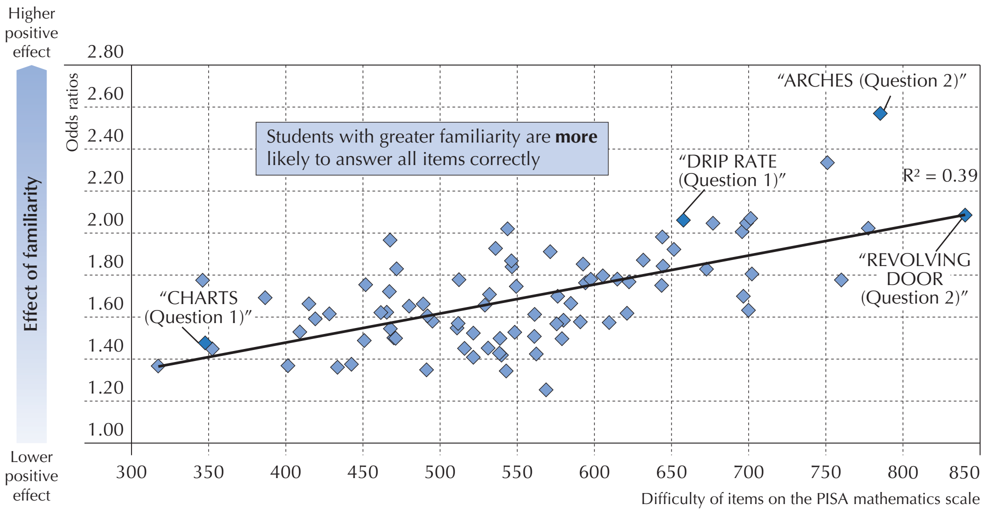
For example, compare the items DRIP RATE Question 1 and REVOLVING DOOR Question 2, both of which show strong associations between familiarity and correct answers. Both are difficult test questions (although not equally difficult) that require students to use their knowledge to solve new problems (see the full text of both tasks at the end of this chapter).

DRIP RATE Question 1 is a task at difficulty Level 5 that requires students to answer a question using a formula $\left(\textit{Drip rate} = \dfrac{dv}{60n}\right)$ that is explicitly stated in the stimulus. REVOLVING DOOR Question 2 is the most difficult task in the assessment, lying at the upper end of Level 6. It asks students to engage in complex geometric reasoning and to perform calculations based on a formula that they should know but that is not recalled in the stimulus. The real problem of designing an efficient revolving door described in the item's stimulus needs to be translated

■ Figure 3.12 ■

Familiarity with mathematics and success on PISA items, by items' difficulty

OECD average (31 countries)



How to read the chart: Values greater than 1 on the vertical axis mean that a one-unit increase in the *index of familiarity with mathematics* increases the probability of answering a given question correctly.

Notes: The OECD countries included in the analysis are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, the Netherlands, New Zealand, Poland, Portugal, Slovenia, the Slovak Republic, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The analysis only includes the items administered to those countries.

The difficulty of the item is set at a 0.62 threshold, meaning that a student who scores 600 in mathematics has a 62% chance of correctly answering an item with a difficulty level of 600.

Source: OECD, PISA 2012 Database, Table 3.12.

StatLink *http://dx.doi.org/10.1787/888933377406*

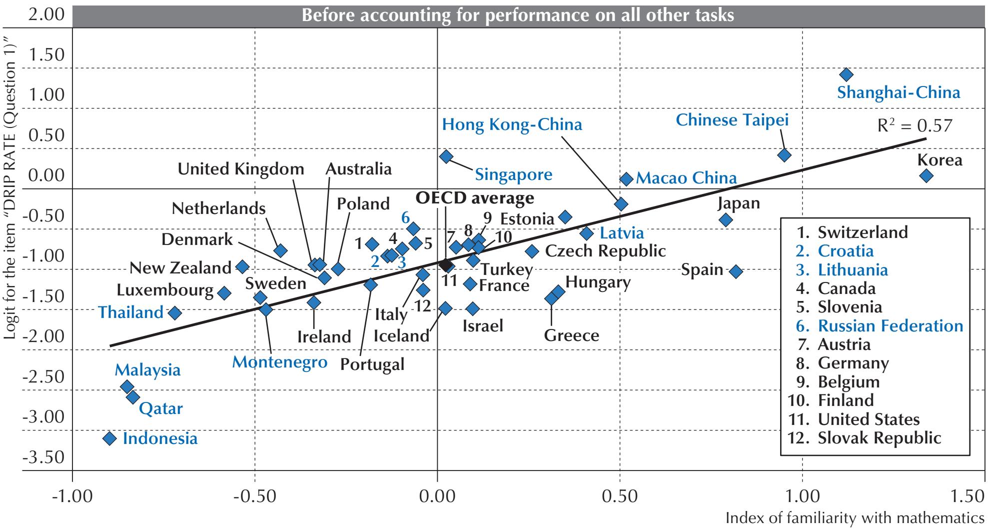
into geometrical terms and back again at multiple points. Only people who are highly skilled in mathematics have the capacity to model a real situation in a mathematical form.

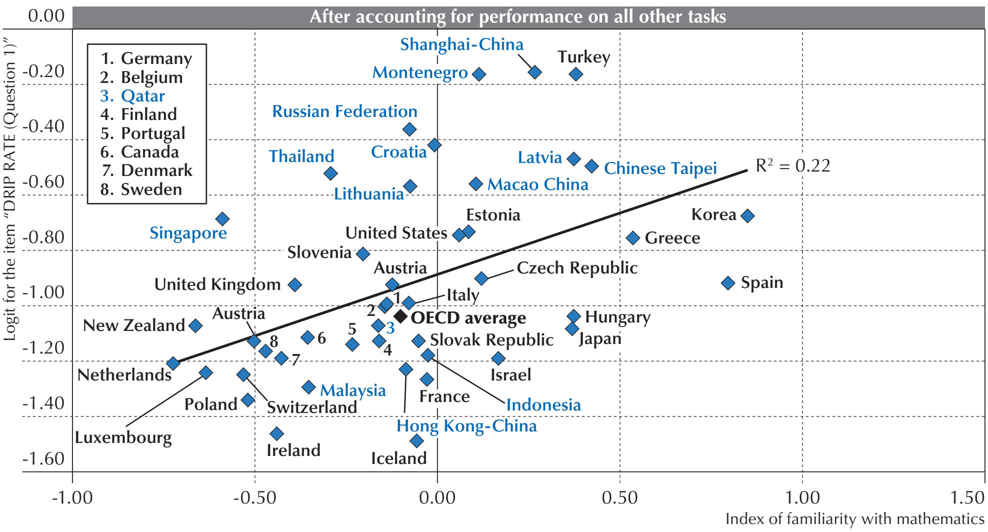
The top panel of Figure 3.13 shows that students with greater familiarity with mathematics are more likely to complete correctly a relatively difficult task, like DRIP RATE Question 1. Between-country differences in familiarity explain 57% of the variation in the correct response rate. This strong relationship between familiarity and performance on DRIP RATE might be due to a causal effect of familiarity on students' capacity to respond correctly, or to the fact that the countries where students perform better in PISA (because of the quality of the teachers, the motivations of students or parents, or other possible reasons) are also the countries with a more challenging mathematics curriculum. Looking at the relationship between solution rates to the DRIP RATE Question 1 task and familiarity *after accounting* for countries' performance on all the other mathematics tasks can help to clarify the direction of causality. Even after taking into account students' performance on all the other tasks in the PISA test, familiarity with mathematics is still positively associated with completing the task DRIP RATE correctly, and explains 22% of the system-level variation in the solution rate (bottom panel of Figure 3.13).

■ Figure 3.13 ■

Familiarity with mathematics and performance on a difficult mathematics task

Country average logit for the PISA Level 5 item "DRIP RATE (Question 1)"





Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
The OECD average is based on 31 countries with available data.
Source: OECD, PISA 2012 Database, Table 3.13.
StatLink http://dx.doi.org/10.1787/888933377412

This means that a higher familiarity with mathematics matters relatively more in explaining performance on a task that requires students to apply their procedural knowledge to a real setting than in other types of tasks.

Similarly, the top panel of Figure 3.14 shows that students in countries with greater familiarity with mathematics are more likely to answer correctly REVOLVING DOOR Question 2. But after accounting for performance on all the other tasks, a country's average level of familiarity with mathematics is not correlated with the percentage of students who answered correctly (bottom panel of Figure 3.14). After taking students' overall mathematics ability into account, greater familiarity is positively associated with the ability to answer the task DRIP RATE correctly but not the item REVOLVING DOOR Question 2, which requires students to engage in more advanced reasoning.

The different relationship between familiarity and performance in the two tasks, before and after accounting for students' overall performance, suggests that familiarity can improve performance in PISA, but only up to a point. The same analysis gives similar results when performed at the student level. After accounting for performance on all the other tasks, familiarity with mathematics is positively related to correct answers to DRIP RATE in 13 of 31 OECD countries and to REVOLVING DOOR in only in 4 of 31 OECD countries (Table 3.15). Frequent exposure to mathematics can make a difference to students trying to tackle problems like DRIP RATE, which states the main terms of the problem and requires students to apply procedures they learned at school. But familiarity with mathematics alone may not be sufficient for solving problems that require the ability to think and reason mathematically, like REVOLVING DOOR.

Developing competence and flexibility to solve demanding problems thus requires both a solid knowledge of mathematics content and extensive practice in searching for creative solutions to mathematics problems. Effective mathematics teachers cover the fundamental elements of the mathematics curriculum and still find the time to expose student to problems that promote conceptual understanding and activate students' cognitive abilities. Recognising mathematics problem solving as one of the ultimate goals of mathematics education, many countries are making specific efforts to develop higher-order thinking skills through the mathematics curriculum: see Box 3.2 for some examples.

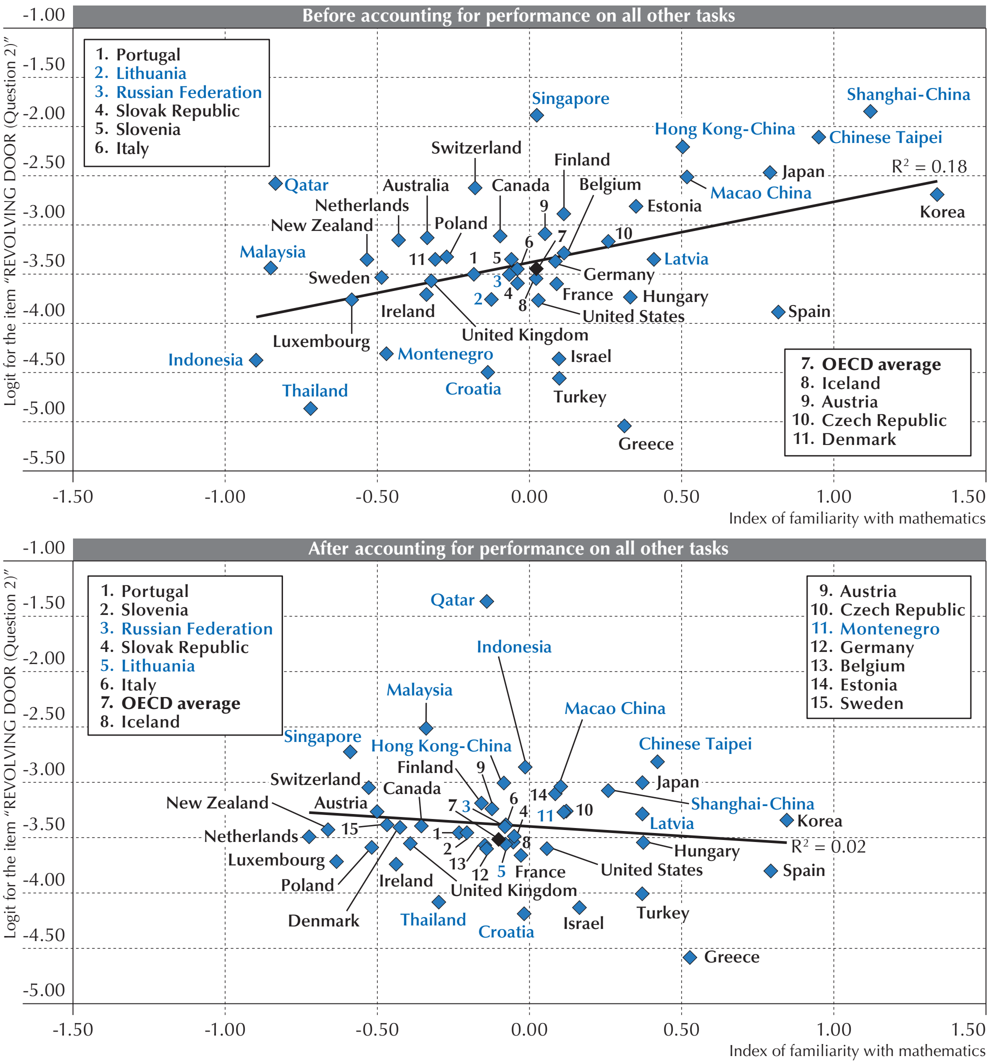
THE LINKS BETWEEN OPPORTUNITY TO LEARN, MATHEMATICS LITERACY AND SOCIO-ECONOMIC STATUS

The analyses presented so far have shown a strong link between opportunity to learn and socio-economic status, and another strong link between opportunity to learn and performance in PISA. Putting the two stories together, how much of the performance gap related to socio-economic status can be explained by the frequency of exposure to mathematics? Figure 3.15 shows that around 19% of the performance difference between socio-economically advantaged and disadvantaged students can be attributed to differences in their familiarity with mathematics, on average across OECD countries (16% after taking into account other student and school characteristics). In Korea, the performance gap related to socio-economic status would be

■ Figure 3.14 ■

Familiarity with mathematics and performance on the most difficult mathematics task

Country average logit for the PISA Level 6 item "REVOLVING DOOR (Question 2)"



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.

The OECD average is based on 31 countries with available data.

Source: OECD, PISA 2012 Database, Table 3.14.

StatLink http://dx.doi.org/10.1787/888933377428

reduced by 29 points (34% of the total) if disadvantaged students had the same familiarity with mathematics as advantaged students have (Table 3.16).

Box 3.2. Integrating higher-order thinking skills in the mathematics curriculum

In addition to covering relevant mathematics contents, a number of countries have recently reformed their mathematics curricula with a view to fostering students' higher-order thinking skills and problem-solving ability. A few examples are reported below.

The national curriculum for mathematics in **England** published in 2013 aims to ensure that all pupils not only become fluent in the fundamentals of mathematics, but also acquire the skills to reason mathematically and to solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication (Department for Education, England, 2013). Also the new Curriculum for Excellence (CfE) in **Scotland** emphasises the development of higher-order skills, such as thinking about complex issues, problem solving, analysis and evaluation; creativity; and critical-thinking skills – making judgements and decisions, developing arguments and solving complex problems (Education Scoltland, 2011).

In **Korea**, teaching and learning problem solving was part of the curriculum since the 1980s. The 2007 revision aimed at further engaging students in mathematical processes such as mathematical reasoning, problem solving, and communication. As a part of the 2007 revision of the lower secondary school curriculum, problem solving was integrated into all mathematics areas (Kim et al., 2012).

Mathematical problem solving is the central priority of **Singapore**'s mathematics framework introduced in the 1990s. The primary aim of the mathematics curriculum is to enable pupils to develop their ability in mathematical problem solving, which includes using and applying mathematics in practical tasks, in real life problems and within mathematics itself (Ginsburg et al., 2005; Ministry of Education, Singapore 2012). In addition, the Teach Less, Learn More (TLLM) initiative launched in 2003 aimed at reducing the curriculum content taught via direct teaching and engage students in more thinking and problem-solving tasks (Berinderjeet et al., 2009).

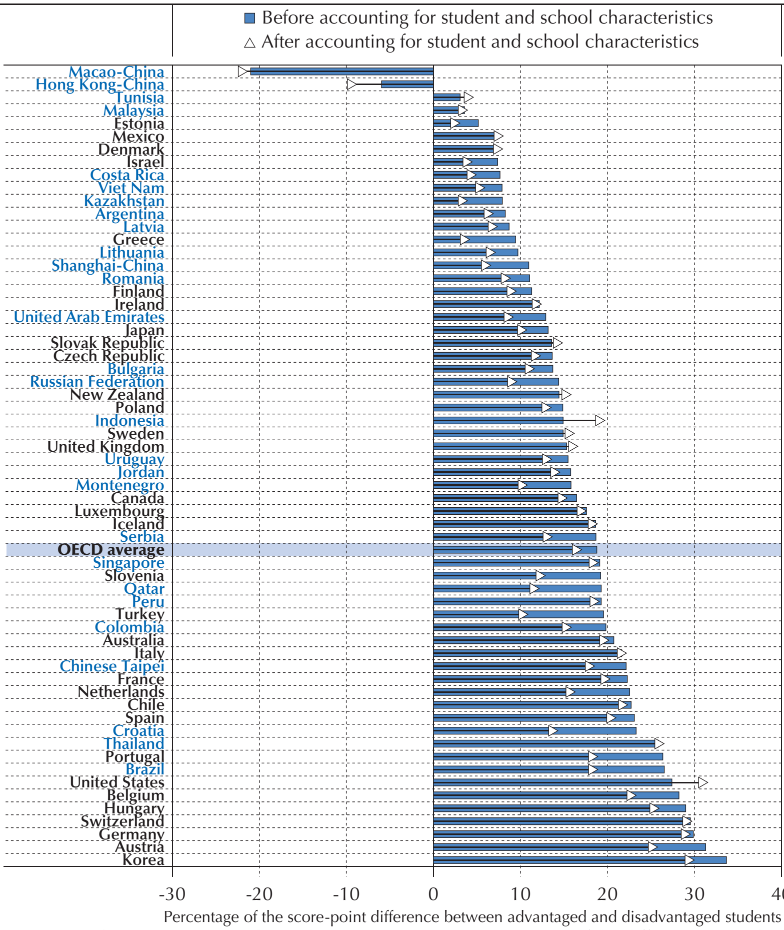
Again, looking at individual tasks in PISA can provide a more fine-grained picture of how opportunity to learn mediates the relationship between socio-economic status and mathematical literacy. Figure 3.16 shows that disadvantaged students lag behind other students across all items, but more so on the most difficult items. On average across OECD countries, a disadvantaged student can be expected to be 23% less likely to solve the easy item CHARTS Question 1 than the average student, but he or she is more than 70% less likely to solve REVOLVING DOOR Question 2.

Figure 3.16 also shows that when disadvantaged students' relative lack of familiarity with mathematics is taken into account, the performance gap related to socio-economic status narrows. But the effect varies, depending on the difficulty of the mathematics problem. For example, the effect is stronger on ARCHES Question 2, a task that mostly requires students to

■ Figure 3.15 ■

Differences in performance related to familiarity with mathematics, by socio-economic status

Percentage of the score-point difference between advantaged and disadvantaged students explained by different familiarity with mathematics



How to read the chart: The OECD average shows that across OECD countries, 19% of the difference in mathematics scores between advantaged and disadvantaged students is explained by disadvantaged students being less familiar with mathematics. This percentage decreases to 16% after accounting for student and school characteristics.

Notes: "Student and school characteristics" include: student's gender, mathematics learning time, whether student's country of birth is different from that in which the test was conducted, rural location of the school, private or public ownership of the school, academic or vocational track, school's selectivity, and indices of teacher support, use of cognitive-activation strategies and disciplinary climate.

Socio-economically advantaged students are defined as those students in the top quarter of the *PISA index of economic, social and cultural status* (ESCS). Disadvantaged students are students in the bottom quarter of ESCS.

In Hong Kong-China and Macao-China, the percentage is negative because disadvantaged students reported greater familiarity with mathematics than advantaged ones. In these economies, eliminating the difference in familiarity would increase the performance gap between advantaged/disadvantaged students.

Countries and economies are ranked in ascending order of the percentage of the performance gap between advantaged and disadvantaged students explained by familiarity with mathematics, before accounting for school characteristics.

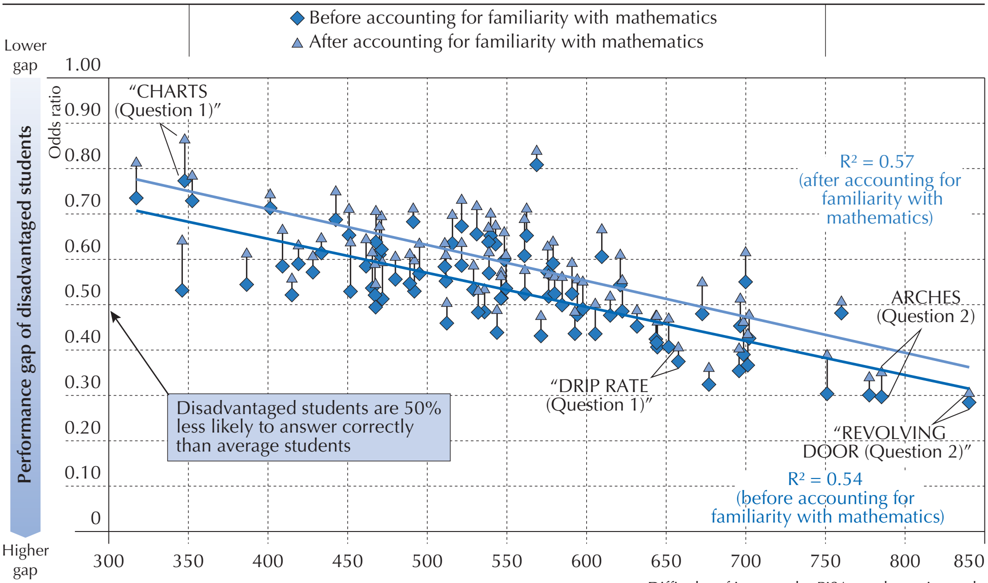
Source: OECD, PISA 2012 Database, Table 3.16.

StatLink http://dx.doi.org/10.1787/888933377436

■ Figure 3.16 ■

Socio-economic status and mathematics performance, by item difficulty

Change in the probability of answering an item correctly associated with socio-economic disadvantage



How to read the chart: A value of 1 on the vertical axis means that disadvantaged students have the same likelihood of answering the item correctly as the average student, while a value of 0.5 means they are 50% less likely to answer correctly. For each item, the distance between the triangle and the diamond reflects the effect of disadvantaged students' lesser familiarity with mathematics on the performance gap between disadvantaged and average students.

Notes: The OECD countries included in the analysis are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, the Netherlands, New Zealand, Poland, Portugal, Slovenia, the Slovak Republic, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The analysis only includes the items administered to those countries.

The difficulty of the item is set at a 0.62 threshold, meaning that a student who scores 600 in PISA mathematics has a 62% chance of answering correctly an item with a difficulty level of 600.

Source: OECD, PISA 2012 Database, Table 3.17.

StatLink *http://dx.doi.org/10.1787/888933377448*

apply procedural knowledge, than on REVOLVING DOOR Question 2, a task that requires students to use of a broader set of mathematics skills.

Which mathematics skills do disadvantaged students have fewer opportunities to develop at school? The PISA mathematics framework defines a set of mathematics competencies that students need to have in order to make use of their knowledge in a variety of contexts (Box 3.3; OECD, 2013a). There is a substantial overlap among these competencies; indeed it is usually necessary to draw on several of them at once to solve a challenging problem.

Box 3.3. **Fundamental mathematical competencies**

The mathematics assessment framework describes a set of fundamental mathematical competencies needed by individuals to make use of their mathematics knowledge and skills (OECD, 2013a). These capabilities are derived from the mathematical competencies described in previous research (Niss and Højgaard, 2011). The framework for the 2012 PISA survey defines the following capabilities:

- **Communication:** This capability involves reading, decoding and interpreting statements, questions, tasks or objects enabling the individual to form a mental model of the situation, as an important step in understanding, clarifying and formulating a problem. Communication may also involve presenting and explaining one's mathematical work or reasoning.
- **Mathematising:** Mathematical literacy can involve transforming a problem defined in the real world to a strictly mathematical form (which can include structuring, conceptualising, making assumptions, and/or formulating a model), or interpreting or evaluating a mathematical outcome or a mathematical model in relation to the original problem.
- **Representation:** This can entail selecting, interpreting, translating between, and using a variety of representations to capture a situation, interact with a problem, or to present one's work. Representations may include graphs, tables, diagrams, pictures, equations, formulae and concrete materials.
- **Reasoning and argument:** This capability involves logically rooted thought processes that explore and link problem elements so as to make inferences from them, check a justification that is given or provide a justification of statements or solutions to problems.
- **Devising strategies for solving problems:** This involves a set of critical control processes that guide an individual to effectively recognise, formulate and solve problems. This skill is characterised as selecting or devising a plan or strategy to use mathematics to solve problems, as well as guiding its implementation.
- **Using symbolic, formal and technical language and operations:** This involves understanding, interpreting, manipulating, and making use of symbolic expressions within a mathematics context (including arithmetic expressions and operations), as well as understanding and using formal constructs based on definitions, rules and formal systems, and using algorithms with these entities.

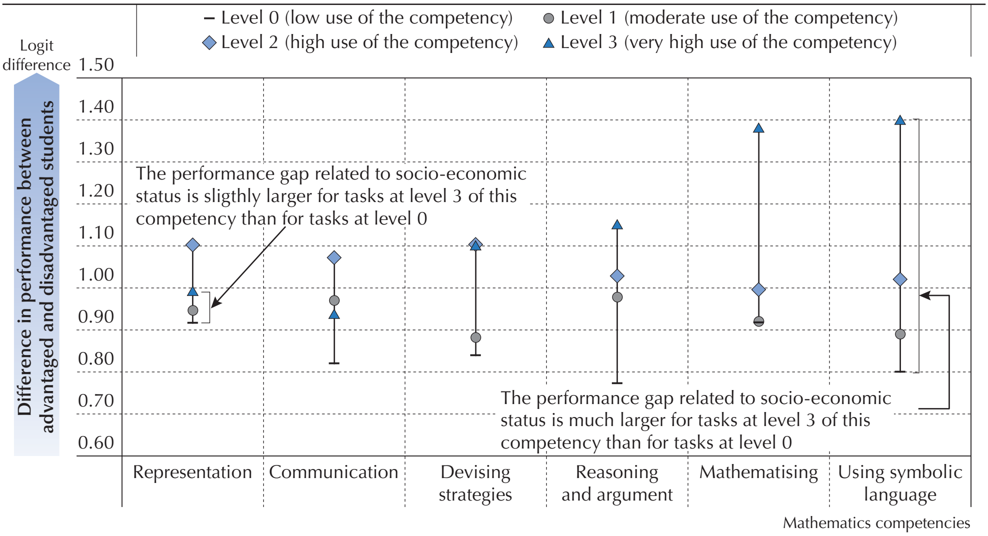
Experts involved in PISA implementation analysed PISA mathematics survey questions and judged the extent to which successfully answering those questions demanded the activation of six mathematical competencies mentioned in the PISA framework. The study involved the operational definition of these competencies and the description of four levels of each competency, recognising some degree of overlapping and interaction across competencies (Turner, 2012). An empirical validation of this classification found that the set of six competencies could predict more than 70% of the variability in item difficulty (Turner and Adams, 2012).

Mathematics experts who were involved in the development of PISA classified the mathematics items according to the type and level of competencies they require (Turner, 2012). Figure 3.17 shows that the performance gap between advantaged and disadvantaged students is significantly wider for those tasks that require greater use of two fundamental competencies: "using symbolic, formal and technical language and operations", and "mathematising", defined as the ability to construct a mathematical model from a real situation, finding a mathematical solution, and interpreting and validating the solution. As Chapter 5 will further discuss, these results suggest that, if the performance gap related to socio-economic status is to be fully closed, disadvantaged students would benefit not only from any policy that increases opportunities for them to develop technical and procedural mathematics skills, but also from more experience with mathematical modelling and using symbolic language.

■ Figure 3.17 ■

Socio-economic status and success on PISA mathematics tasks, by required mathematics competencies

Logit differences between advantaged and disadvantaged students according to the level of competency required by the task, OECD average

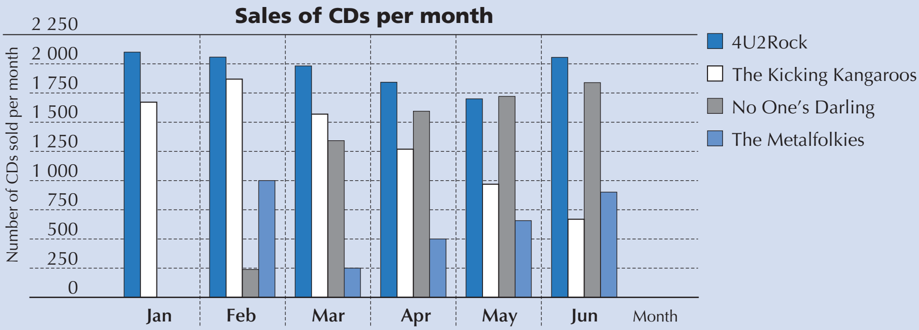


Notes: Socio-economically advantaged students are defined as those in the top quarter of the *PISA index of economic, social and cultural status* (ESCS). Disadvantaged students are those in the bottom quarter of ESCS.
The classification of PISA mathematics items by type and level of competency required is drawn from Turner (2012).
A logit is the logarithm of the odds of answering correctly. A logit value of 0 means that 50% of respondents answered the question correctly. Higher/lower logits correspond to higher/lower rates of correct responses and to lesser/greater item difficulty.
All values are statistically significant.
Source: OECD, PISA 2012 Database, Table 3.18.
StatLink *http://dx.doi.org/10.1787/888933377455*

EXAMPLES OF PISA MATHEMATICS UNITS

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



CHARTS – *QUESTION 1*

How many CDs did the band *The Metalfolkies* sell in April?

A. 250

B. 500

C. 1 000

D. 1 270

Scoring

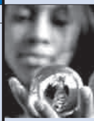
Description: Read a bar chart
Mathematical content area: Uncertainty and data
Context: Societal
Process: Interpret

Full Credit

B. 500

No Credit

Other responses.
Missing.



DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, *D*, in drops per minute for infusions.

They use the formula $D = \dfrac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

DRIP RATE – *QUESTION 1*

A nurse wants to double the time an infusion runs for.

Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

....

....

....

Scoring

Description: Explain the effect that doubling one variable in a formula has on the resulting value if other variables are held constant
Mathematical content area: Change and relationships
Context: Occupational
Process: Employ

Full Credit

Explanation describes both the direction of the effect and its size.

- It halves
- It is half
- D will be 50% smaller
- D will be half as big

Partial Credit

A response which correctly states EITHER the direction OR the size of the effect, but not BOTH.

- D gets smaller *[no size]*
- There's a 50% change *[no direction]*
- D gets bigger by 50% *[incorrect direction but correct size]*

No Credit

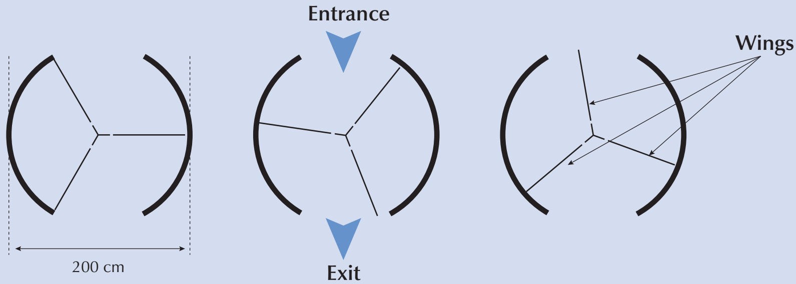
Other responses.

- D will also double *[both the size and direction are incorrect.]*

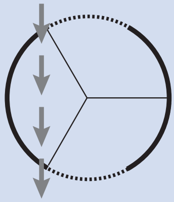
Missing.

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



REVOLVING DOOR – *QUESTION 2*



The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: ....cm

Scoring

Description: Interpret a geometrical model of a real life situation to calculate the length of an arc
Mathematical content area: Space and shape
Context: Scientific
Process: Formulate

Full Credit

Answers in the range from 103 to 105. *[Accept answers calculated as 1/6th of the circumference (100 π/3). Also accept an answer of 100 only if it is clear that this response resulted from using π = 3. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]*

No Credit

Other responses.

- 209 *[states the total size of the openings rather than the size of "each" opening]*

Missing.

Notes

1. This information is reported by representatives of countries/economies who responded to the TIMSS Eight Grade Curriculum Questionnaire. The full set of data is available at http://timss.bc.edu/timss2011/international-database.html.

2. The logit transformation of the percentage of students who responded correctly to the item takes into account the non-linear relationship between answering questions correctly and the items' difficulty (OECD, 2013c). A logit value of 0 means that 50% of respondents answered the question correctly; positive logits mean higher rates of correct answers and negative logits mean lower rates of correct responses.

3. Average difficulties by content area displayed in Figure 3.2 are based the following number of PISA test items: change and relationships: 8 items; quantity: 10 items; space and shape: 9 items; uncertainty and data: 7 items.

4. To understand how the results in Figure 3.5 are derived, consider the following two equations relating PISA average scores and learning time for students in each school and grade:

$$Score_{Mg} = \beta\ Hours_{Mg} + v_g + u_{Mg} \tag{1}$$

$$Score_{Rg} = \beta\ Hours_{Rg} + v_g + u_{Rg} \tag{2}$$

Where the subscripts *R* and *M* indicate students' averages in PISA reading and mathematics, respectively, the subscript *g* indicates the average for all students in the same grade within the same school, v_g represents characteristics of schools and grades that do not vary across subjects of instruction, and u is an error term.

Taking the difference of (1) and (2), gives:

$$\Delta Score_{Mg-Rg} = \hat{\beta}\ \Delta Hours_{Mg-Rg} + \Delta u_{Mg-Rg} \tag{3}$$

As can be seen, the first-difference (fixed-effect) regression in (3) estimates the relation between learning time and PISA scores ($\hat{\beta}$)accounting for subject-invariant differences across schools and grades (the term v_g gets cancelled out when differencing equations 1 and 2).

5. The *index of disciplinary climate* summarises students' reports on the frequency of noise, disorder and inactivity due to disciplinary issues in the classroom.

6. The results in Figure 3.11 are derived similarly to those shown in Figure 3.5 (see previous endnote). Consider the following two equations relating PISA average scores and exposure to pure mathematics for students in two contiguous grades:

$$Score_{0S} = \beta\ Exposure_{0S} + v_S + u_{0S} \tag{1}$$

$$Score_{1S} = \beta\ Exposure_{1S} + v_S + u_{1S} \tag{2}$$

Where the subscripts *0* and *1* indicate averages for students attending school *s* in grades *0* and *1*, respectively, the subscript v_S represents characteristics of schools that do not vary across grades, and u is an error term.

Taking the difference of (2) and (1), gives:

$$\Delta Score_{1-0S} = \hat{\beta}\ \Delta Exposure_{1-0S} + \Delta u_{1-0S} \tag{3}$$

As can be seen, the first-difference (fixed-effect) regression in (3) estimates the relation between exposure to pure mathematics and PISA scores ($\hat{\beta}$) accounting for grade-invariant differences across schools (the term v_S gets cancelled out when differencing equations 1 and 2).

7. The item-level analysis presented in this chapter is restricted to paper-based items because these are common to the largest number of countries.

8. The item, ARCHES Question 2, is not included in the examples at the end of this chapter because it is not a released item.

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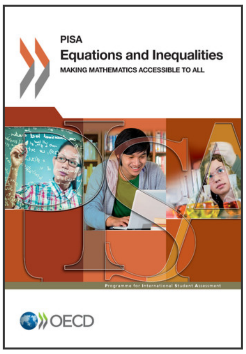

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