2

Variations in Students' Exposure to and Familiarity with Mathematics

Students' exposure to mathematics varies within countries even more than between countries. This chapter first explores how access to mathematics content varies by socio-economic status and other student characteristics, such as gender, immigrant background and attendance at pre-primary school. It then analyses the extent to which school- and system-level factors – including student sorting and teaching resources and practices – can produce segregation in opportunities to learn mathematics based on students' socio-economic status.

The statistical data for Israel are supplied by and under the responsibility of the relevant Israeli authorities. The use of such data by the OECD is without prejudice to the status of the Golan Heights, East Jerusalem and Israeli settlements in the West Bank under the terms of international law.

Lack of access to mathematics content at school can leave young people socially and economically disadvantaged for life. Who gets to learn mathematics, and the nature of the mathematics that is learned, have an impact on education systems, social cohesion and productivity. Education systems that fail to provide the same opportunities to all students can end up reinforcing, rather than beginning to dismantle, social inequalities. When education is no longer a pathway to individual fulfilment and social mobility, talent is wasted and feelings of injustice grow. Failing disadvantaged youth at school can also pave the way for a wide range of social problems later on, including poverty, poor health and crime (Schoenfeld, 2002; OECD, 2012).

This chapter focuses on how opportunities to learn mathematics vary within countries. Across all education systems, socio-economically disadvantaged young people have less access to mathematics content. The results confirm the findings of extensive research on the strong links between socio-economic advantage, mastery of mathematics and perseverance in secondary school mathematics (Crosnoe and Schneider, 2010).

Students of higher socio-economic status have an advantage from the beginning: they tend to have access to high-quality learning opportunities, both formal and informal. At an early age, they often attend better resourced and better organised elementary schools, and they have access to after-school programmes that enrich their learning experience as they approach high school (Downey, von Hippel and Broh, 2004; Entwisle, Alexander and Olson, 2005; Lareau, 2011; NICHD Early Child Care Network, 2005). The parents of these students tend to have greater experience navigating through the education system, which makes it more likely that their children will pursue higher education and succeed in the labour market (Morgan, 2005).

What the data tell us

- Across OECD countries, around 9% of the variation in familiarity with mathematics within countries is explained by students' socio-economic status and by the concentration of socio-economic advantage in certain schools. Socio-economic differences among students and schools account for less than 1% of the variation in Estonia and Malaysia and for more than 20% of the variation in Hungary and Liechtenstein.
- Around 70% of the students who have at least one tertiary-educated parent reported that they know well or have often heard of the concept of linear equation; only 52% of students whose parents have only primary education as their highest level of attainment so reported.
- Around 54% of the variation among OECD countries in the impact of students' and schools' socio-economic status on students' familiarity with mathematics is explained by system-level differences in the age at which students are tracked into vocational or general/academic programmes or schools.
- Ability grouping is more prevalent in disadvantaged schools than in advantaged schools, on average across OECD countries.
- On average across OECD countries, teachers' use of cognitive-activation strategies is associated with greater familiarity with mathematics among students in socio-economically advantaged schools; but this is not the case in disadvantaged schools.

PISA data show large variations across countries in the association between socio-economic status (Box 2.1) and students' familiarity with mathematics, suggesting that the organisation of education systems can either mitigate or reinforce inequalities in access to knowledge.[1] The mechanisms in place for selecting students in schools according to their perceived ability and preparation seem to play a significant role in reducing opportunities to learn mathematics among disadvantaged students.

What these results mean for policy

- As exposure to, and familiarity with, mathematics are strongly correlated with students' socio-economic status, education systems and policies should be designed with the aim of giving all students equal opportunity to learn mathematics concepts and to practice challenging mathematics tasks.
- In order to give all students equal opportunity to learn mathematics, tracking should be delayed and/or struggling students should be offered individualised instruction tailored to their needs.
- More professional training in teaching in multicultural settings should be provided to teachers, particularly to those who work in disadvantaged schools.
- All students would benefit from teaching practices that emphasise mathematics reasoning and problem solving; but policy makers, school authorities and teachers should ensure that such practices do not take time away from covering key mathematics concepts, especially for socio-economically disadvantaged students.

VARIATIONS IN ACCESS TO MATHEMATICS CONTENT WITHIN COUNTRIES

Catering to the needs of a diverse student body and ensuring consistently high standards across schools represent formidable challenges for any school system (OECD, 2004). Variations in opportunity to learn mathematics within countries can be related to a variety of factors, including regional differences in the socio-economic and cultural characteristics of the communities that are served by schools; the quality of the school staff or the education policies implemented in some schools and not in others; the distribution of human and financial resources available to schools; and system-level factors, such as the way students are grouped, according to their academic potential and interests, in specific programmes (OECD, 2013a).

Figure 2.1 shows large international differences in the extent to which students' self-reported knowledge of mathematics varies within each country. The total length of the bars indicates the observed variation in the *index of familiarity with mathematics*. The variation in familiarity with mathematics is more than four times greater in Liechtenstein and Spain than in Indonesia. Across OECD countries, around 86% of the country-level variation in familiarity with mathematics can be traced to differences across students who attend the same school, while around 14% can be ascribed to differences across students who attend different schools. In Austria, Germany, Hungary, Liechtenstein and Qatar, at or over a quarter of the variation is due to differences across schools. Denmark, Finland, Malaysia, Sweden and Tunisia are the most comprehensive systems, where there is less than 5% variation in students' familiarity with mathematics observed among schools.[2]

Box 2.1. What is socio-economic status and how it is measured in PISA?

Socio-economic status in PISA is a broad concept that summarises many different aspects of a student, school or system. A student's socio-economic status is estimated by an index, the *PISA index of economic, social and cultural status* (ESCS), which is based on such indicators as parents' education and occupation, the number and type of home possessions that are considered proxies for wealth, and the educational resources available at home. The index is built to be internationally comparable (see the *PISA 2012 Technical Report*, OECD, 2014a). Students are considered socio-economically advantaged if they are among the 25% of students with the highest ESCS in their country or economy; socio-economically disadvantaged students are those among the 25% of students with the lowest ESCS. Schools are defined as socio-economically advantaged (disadvantaged) if the average ESCS of students in the school is statistically significantly above (below) that of the average school.

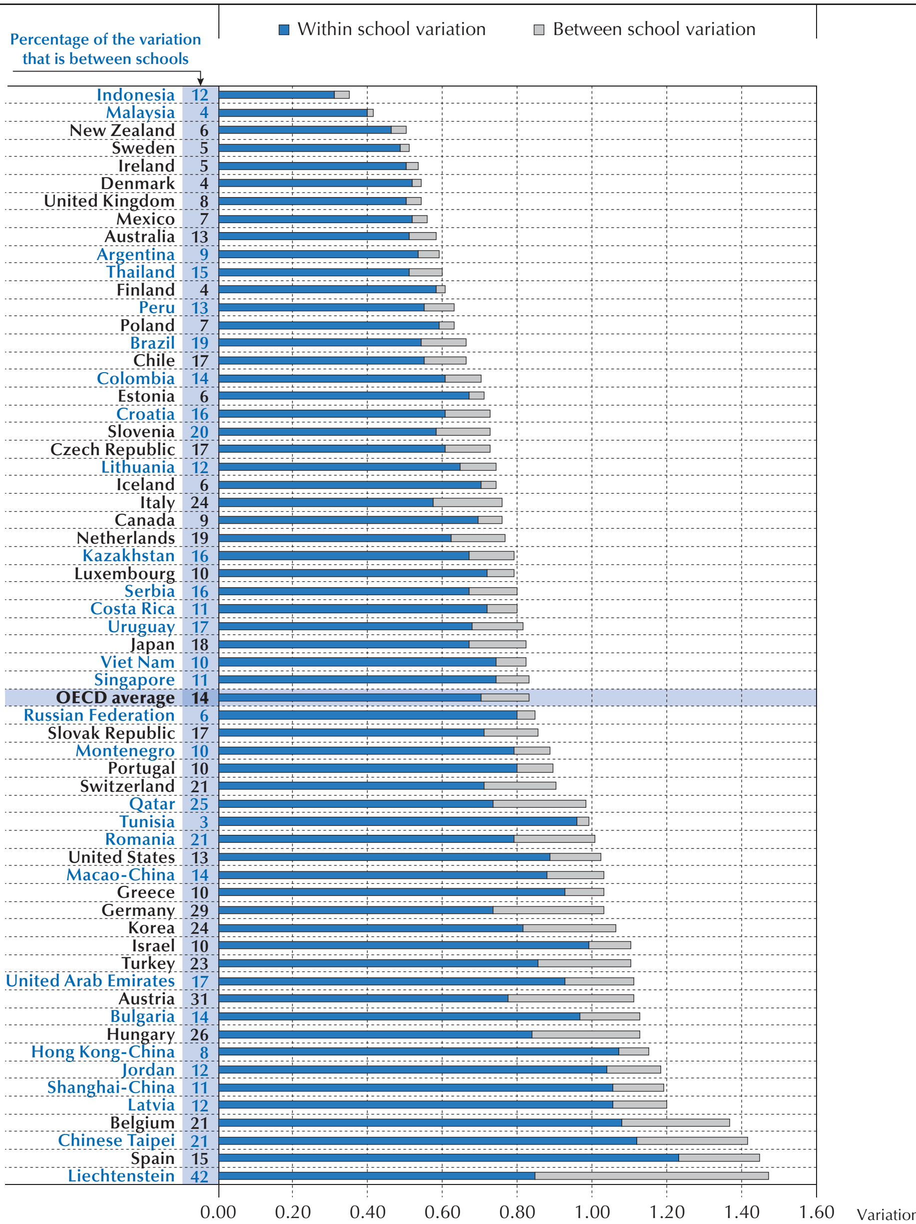
PISA consistently finds that socio-economic status is associated with performance at system, school and student levels. These patterns reflect, in part, the inherent advantages in resources that relatively high socio-economic status provides. However, they also reflect other characteristics that are associated with socio-economic status but that are not measured by the PISA index. For example, at the system level, high socio-economic status is related to greater wealth and higher spending on education. At the school level, higher socio-economic status is associated with a range of characteristics of a community that might be related to student performance, such as a safe environment and the availability of quality educational resources, such as public libraries or museums. At the individual level, socio-economic status may be related to parents' attitudes towards education, in general, and to their involvement in their child's education, in particular.

Source:
OECD (2013a).

Across OECD countries, around 4% of the variation in students' familiarity with mathematics within countries is explained by the socio-economic status of students; this percentage more than doubles when also taking into account the socio-economic composition of the schools – that is, the concentration of students with a similar socio-economic status who attend the same schools (Figure 2.2).[3] The cumulative effect of a student's socio-economic status and the concentration of socio-economic advantage in a school is particularly large in Austria, Hungary and Liechtenstein. In Portugal and Spain, the socio-economic profile of a school adds little to the effect related to an individual student's socio-economic status, suggesting that disadvantaged students lag behind other students in access to mathematics no matter which school they attend. The opposite pattern is observed in the Czech Republic, the Netherlands and Japan, where segregation by socio-economic status occurs mostly between schools.

■ Figure 2.1 ■

Variation in familiarity with mathematics, within and between schools



Note: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Countries and economies are ranked in ascending order of the total variation (within and between) in the index of familiarity with mathematics.

Source: OECD, PISA 2012 Database, Table 2.1.

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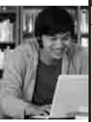
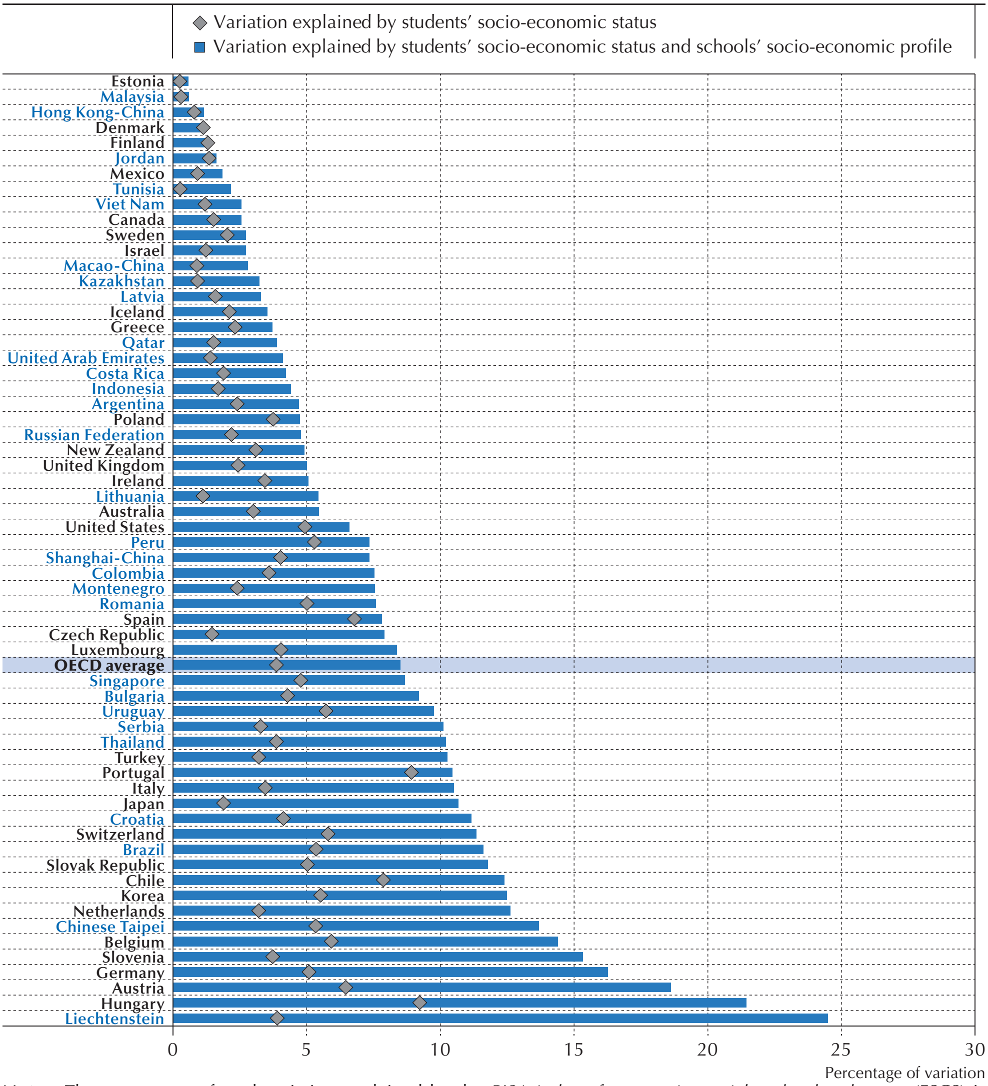


■ Figure 2.2 ■

Variation in familiarity with mathematics explained by students' and schools' socio-economic profile



Notes: The percentage of total variation explained by the *PISA index of economic, social and cultural status* (ESCS) is estimated through a linear model. The relationship between familiarity with mathematics and ESCS is statistically significant in all countries and economies.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).

Estimates for France based on the schools' socio-economic profile were deleted at the request of the country. Students' socio-economic status explains 6.7% of the variation in familiarity with mathematics within the country.

Countries and economies are ranked in ascending order of the percentage of variation in familiarity with mathematics explained by students' socio-economic status and by schools' socio-economic profile.

Source: OECD, PISA 2012 Database, Table 2.2.

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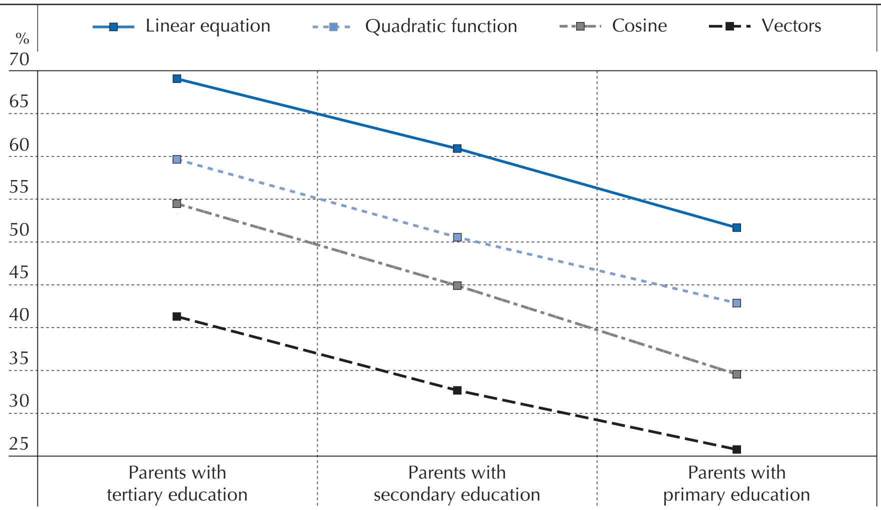
Figure 2.3 shows the magnitude of the differences in familiarity with mathematics associated with students' socio-economic status. Around 70% of the students who have at least one tertiary educated parent reported that they know well or have often heard of the concept of linear equation, on average across OECD countries; only 52% of students whose parents have only primary education as their highest level of attainment so reported. Similarly, around 55% of students with highly educated parents and only 35% of students with low-educated parents reported that they know well or have frequently been exposed to the geometric concept of cosine.

■ Figure 2.3 ■

Familiarity with mathematics concepts, by parents' highest level of education

Percentage of students who reported that they know well or have often heard the concept, OECD average



Source: OECD, PISA 2012 Database, Table 2.5a.

StatLink *http://dx.doi.org/10.1787/888933376994*

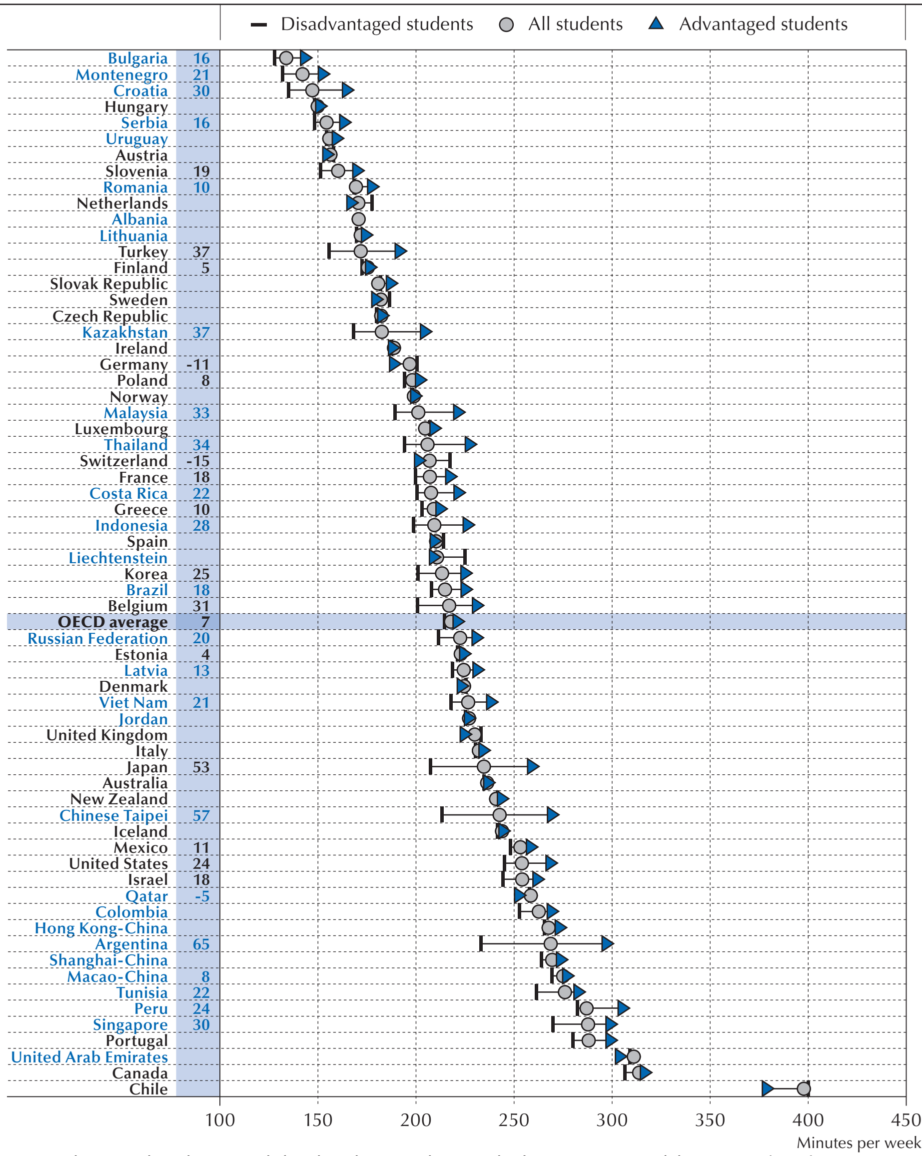
This gap in opportunities to learn mathematics is not strongly related to the time students spend in mathematics classes (Figure 2.4). In fact, in 2012, disadvantaged students spent only seven minutes less per week in mathematics courses at school than advantaged students did (equivalent to one-tenth of a standard deviation), on average across OECD countries. There were only a few exceptions: in Argentina, Japan and Chinese Taipei, disadvantaged students spent around one hour less in mathematics classes per week than advantaged students did.

Rather than the amount of time spent on mathematics, it is how that time is used that influences the difference in familiarity with mathematics between advantaged and disadvantaged students. Figures 2.5a and 2.5b show clearly that disadvantaged students have less exposure to both the applied and the pure mathematics tasks included in PISA (see Chapter 1 for the definition of these indices). In Iceland, Jordan, Korea, New Zealand and Chinese Taipei, the difference in exposure to applied mathematics tasks between advantaged and disadvantaged students is more than 0.5 units (equivalent to half of a standard deviation of the OECD average) (Figure 2.5a).

■ Figure 2.4 ■

Mathematics learning time at school, by students' socio-economic status

Average minutes per week of mathematics instruction in class



Notes: Disadvantaged students are defined as those students in the bottom quarter of the *PISA index of economic, social and cultural status* (ESCS). Advantaged students are students in the top quarter of ESCS.

Only statistically significant differences in mathematics learning time between advantaged and disadvantaged students are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the minutes per week spent learning mathematics at school for all students.

Source: OECD, PISA 2012 Database, Table 2.3.

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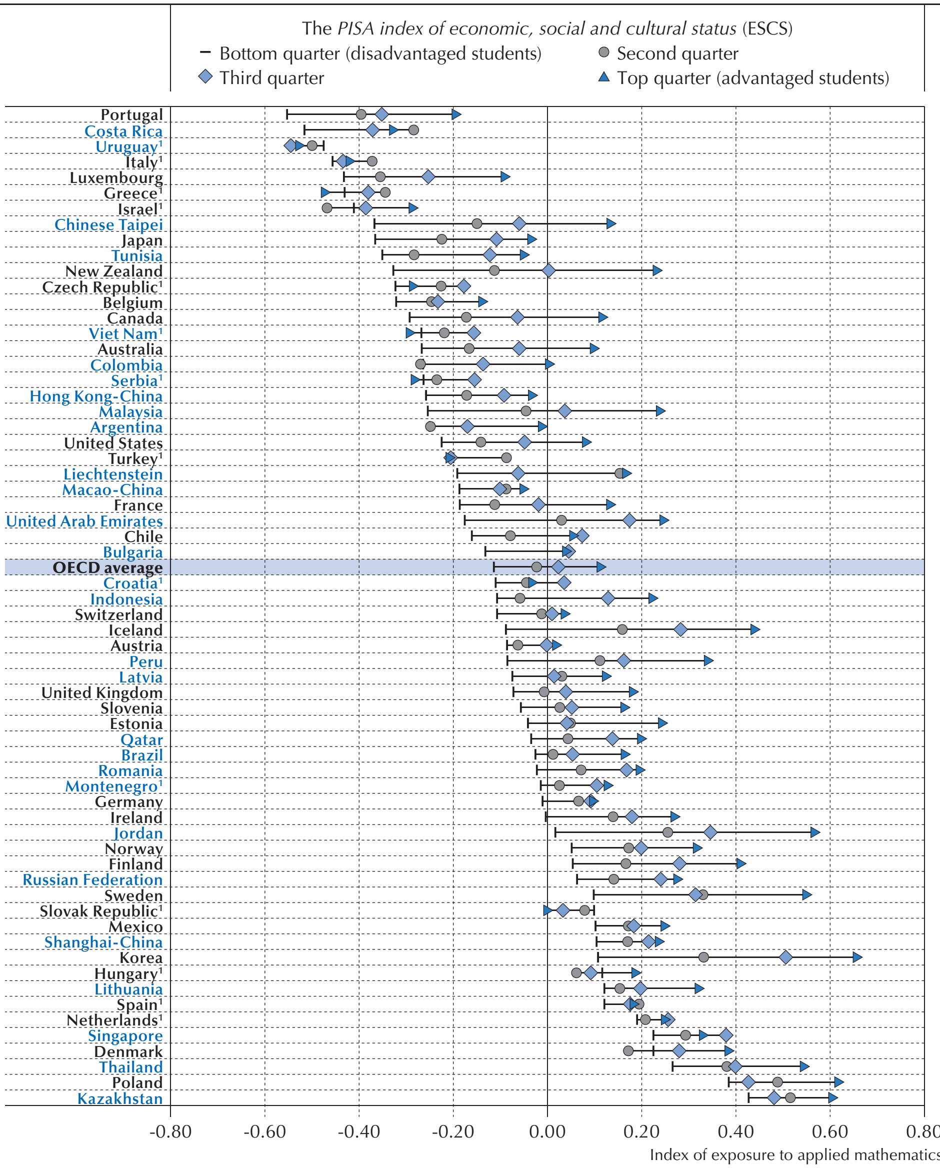
■ Figure 2.5a ■

Exposure to applied mathematics, by students' socio-economic status



1. The difference between the top and the bottom quarters of the *PISA index of economic, social and cultural status* (ESCS) is not statistically significant.

Note: The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

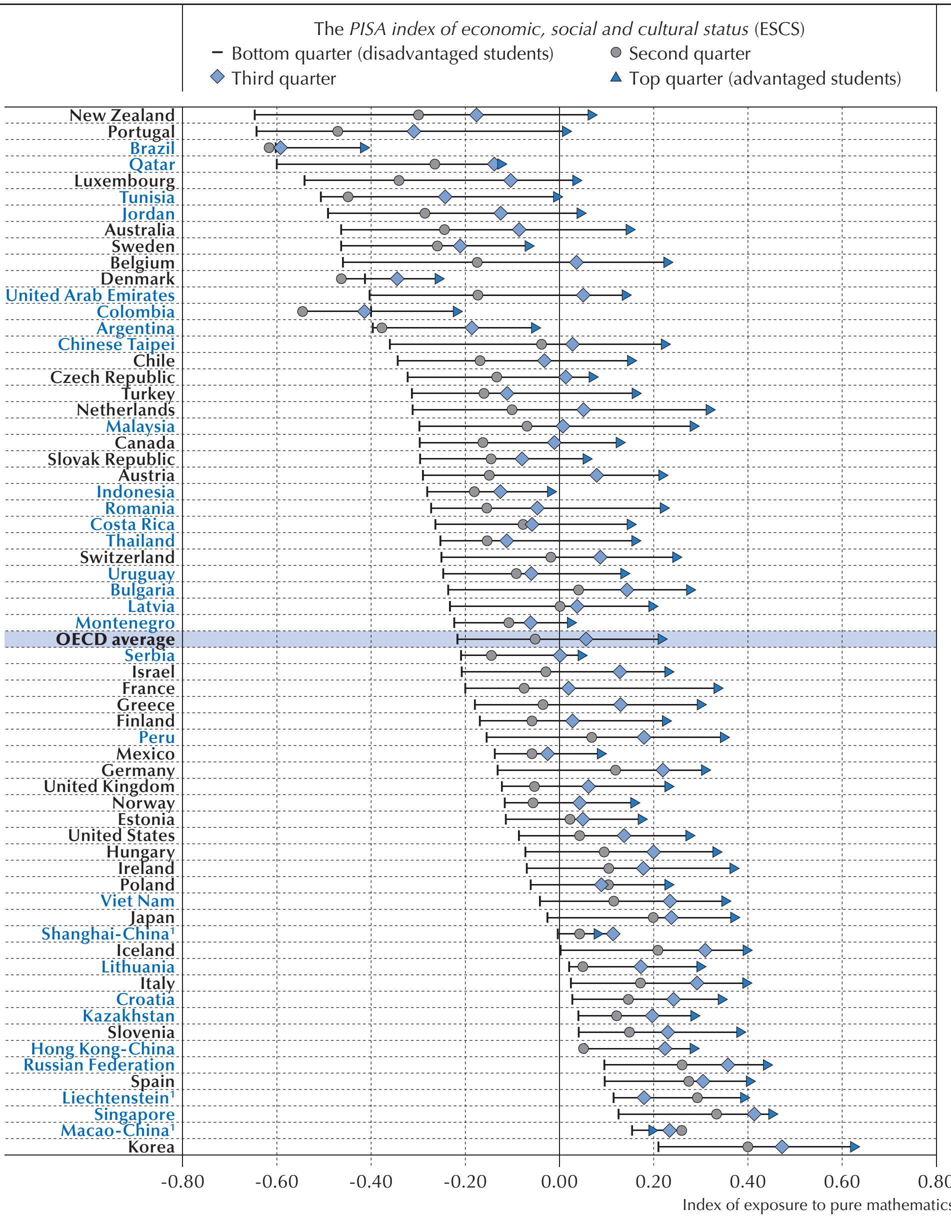
Countries and economies are ranked in ascending order of the average index of exposure to applied mathematics *of students in the bottom quarter of ESCS.*

Source: OECD, PISA 2012 Database, Table 2.4a.

StatLink http://dx.doi.org/10.1787/888933377010

■ Figure 2.5b ■

Exposure to pure mathematics, by students' socio-economic status



1. The difference between the top and the bottom quarters of ESCS is not statistically significant.
Note: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).
Countries and economies are ranked in ascending order of the average index of exposure to pure mathematics *of students in the bottom quarter of ESCS.*
Source: OECD, PISA 2012 Database, Table 2.4a.
StatLink http://dx.doi.org/10.1787/888933377022

The difference in exposure to pure mathematics tasks (functions and equations) is even larger and is statistically significant in all countries and economies except Macao-China, Liechtenstein and Shanghai-China (Figure 2.5b). On average across OECD countries, there is a difference of 0.44 of a standard deviation in the *index of exposure to pure mathematics* between advantaged and disadvantaged students. To put this magnitude in perspective, the average difference between students in the modal grade and students one grade below is 0.29 of a standard deviation (Chapter 1, Table 1.10). In Belgium and New Zealand, the difference in the *index of exposure to pure mathematics* between advantaged and disadvantaged students is over two-thirds of a standard deviation. In 49 of 63 countries and economies with available data, socio-economically disadvantaged students had less than the OECD average exposure to pure mathematics (Figure 2.5b).

These data raise questions about the effectiveness of the time disadvantaged students spend studying mathematics at school. Given a similar investment of time, disadvantaged students still reported having less knowledge about key mathematics concepts, spending less time solving equations and engaging less in relatively simple applied mathematics tasks. What do these students do and learn during the many hours they spend in mathematics classes? Can the knowledge gap be traced to other student characteristics, or is it more strongly linked to how schools and school systems are organised and how they teach mathematics?

INDIVIDUAL STUDENT CHARACTERISTICS AND ACCESS TO MATHEMATICS CONTENT

Gender differences in opportunity to learn mathematics

In most countries, mathematics and mathematics-related fields are indisputably male-dominated. There is no innate reason why girls should not be able to do as well as boys in mathematics; most empirical studies find no gender difference in standardised mathematics scores upon entry to school (Fryer and Levitt, 2010). However, in most of the countries and economies that participate in PISA, girls do worse than boys in mathematics, on average, particularly among high-performing students (OECD, 2014b). Differences in perceived ability in mathematics and mathematics anxiety are major factors behind the gender gap in mathematics performance, and have been shown to predict later achievement and occupational choices (Chapter 4, [Bandura et al., 2001; Dweck, 2007; Eccles, 2007; Häussler and Hoffmann, 2002]).

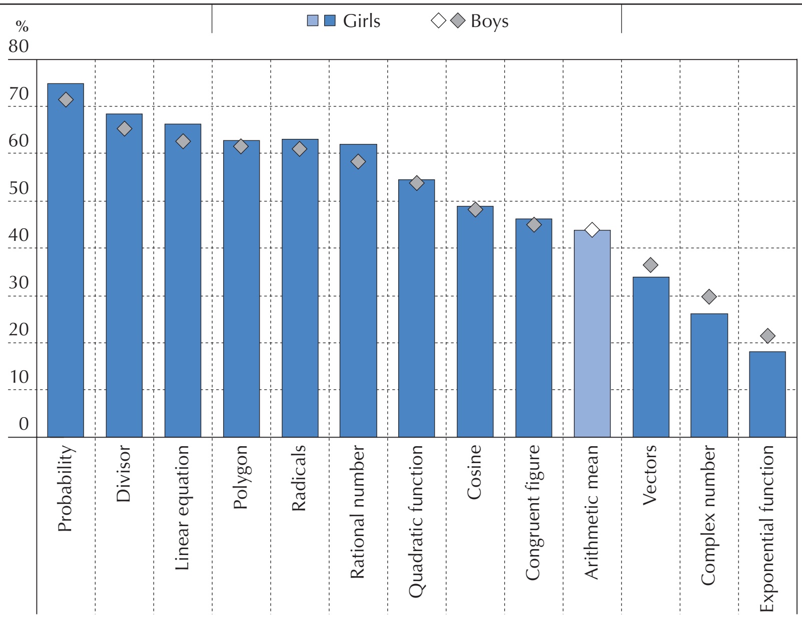
Gender disparities in mathematics achievement might also result from differences in the opportunities boys and girls have to practice their mathematics skills. PISA data show that boys and girls have different opportunities to develop mathematics skills outside of school. For example, girls are less likely than boys to play chess, program computers, take part in mathematics competitions, or do mathematics as an extracurricular activity (OECD, 2015a).

What about opportunities to learn mathematics during school time? Figure 2.6 shows that girls were more likely than boys to report that they often heard of and/or know well most mathematics concepts, even though gender differences are not large in most countries. Girls are more likely than boys to report being familiar with mathematics, particularly with concepts to which most 15-year-olds have been exposed. For example, on average across OECD

■ Figure 2.6 ■

Familiarity with mathematics concepts, by gender

Percentage of students who reported that they know well or have often heard the concept, OECD average



Note: Statistically significant gender differences are marked in a darker tone.
Countries and economies are ranked in descending order of boys' familiarity with the concept.
Source: OECD, PISA 2012 Database, Table 2.5b.
StatLink *http://dx.doi.org/10.1787/888933377036*

countries, 75% of girls and 71% of boys reported a high level of familiarity with the concept of probability. This difference in favour of girls is over 15 percentage points in Jordan, Thailand and the United Arab Emirates (Table 2.5b). By contrast, 30% of boys and 26% of girls are familiar with the more advanced concept of complex numbers. Boys' advantage in familiarity with this concept is over 10 percentage points in Germany, Liechtenstein and Luxembourg. These patterns of exposure reflect the broader picture of gender differences in mathematics, with boys excelling at the top and struggling at the bottom of the performance distribution.

While girls appear to be more likely than boys to have encountered pure mathematics tasks, such as solving quadratic and linear equations, gender differences in self-reported experience with applied mathematics tasks are generally small; in fact, in the large majority of countries and economies there is no difference in boys' and girls' exposure to such tasks (OECD, 2015a). Differences in girls' and boys' likelihood to repeat grade or be enrolled in a vocational rather than an academic programme explain only a small part of the gender differences in students' familiarity with mathematics concepts and with pure mathematics tasks (OECD, 2015a).

Immigrant students' familiarity with mathematics concepts

In most PISA-participating countries and economies, foreign-born students score lower in mathematics than students without an immigrant background, and students who were born in the country in which they sat the PISA test, but whose parents were born outside the country, perform somewhere between the two (OECD, 2015b). On average across OECD countries, immigrant students are 1.7 times more likely than students without an immigrant background to perform in the bottom quarter of the mathematics performance distribution (OECD, 2013a). The performance gap between the two groups of students tends to be smaller in mathematics than in reading, suggesting that language comprehension is one of the most serious hurdles for immigrant students.

Immigrant students are much less familiar with the mathematics concepts that they are expected to learn in secondary school (linear equations, exponential functions, divisors and quadratic functions) than students without an immigrant background. The gap in the self-reported familiarity with mathematics concepts between foreign-born students and students without an immigrant background is particularly large (more than half a standard deviation) in Italy and Spain, two of the OECD countries that saw the largest increase in immigration over the past decade (Table 2.7). In most countries, students who were born in the country in which they sat the PISA test, but whose parents weren't, reported greater familiarity with mathematics than students who were not born in the country. This suggests that late arrival might reduce the opportunities to learn mathematics content, or increase the mismatch between what was learned in the country of origin and what is learned in the destination country.

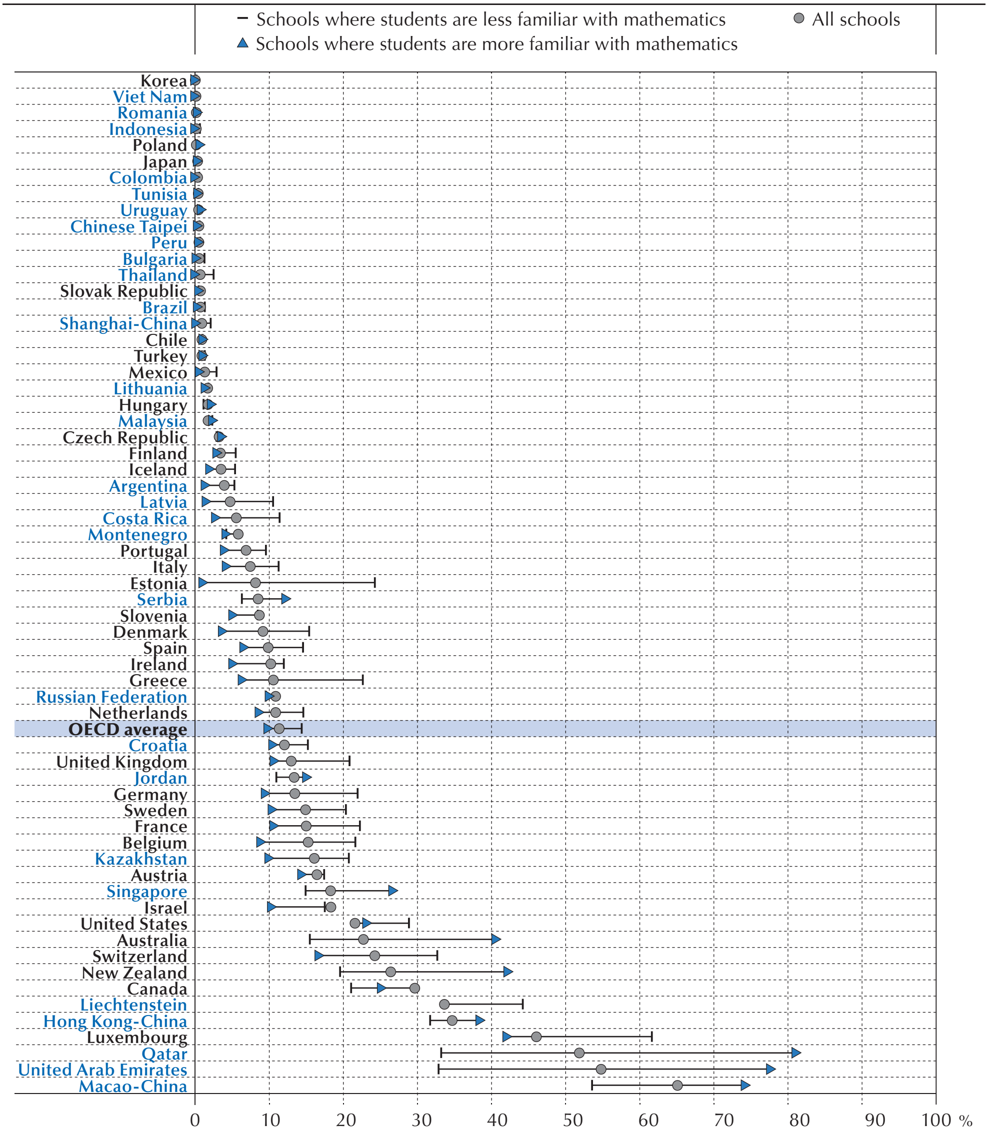
Differences in the quality of instruction and in the depth and coverage of curricula across countries of origin and destination can lead to gaps in students' readiness to learn advanced mathematics material. Immigrant students, and particularly refugees, are also likely to have spent considerable time out of school as they were making their way from their country of origin to their host country. At least one in six immigrant students who attend school in an OECD country lost more than two months of school at least once in his or her life (OECD, 2015b). But apart from these differences, the high concentration of immigrant students in disadvantaged schools in host countries might explain why these students are not familiar with certain mathematics concepts. Immigrant students are often concentrated in schools that suffer from high turnover rates among teachers, less effective learning time, and low-quality educational resources (OECD, 2013a). In these contexts, immigrant students are less likely to overcome their initial learning disadvantages.

Figure 2.7 shows that immigrant students tend to be concentrated in schools where students reported less exposure to mathematics concepts. On average across OECD countries, 14% of students in schools whose students reported relatively less familiarity with mathematics are immigrant students, as are 10% of the students in schools with greater average familiarity with mathematics. In Greece, almost 1 in 4 students in schools where the reported familiarity with mathematics concepts is low is an immigrant student, as is only 1 in 16 students in schools with greater average familiarity with mathematics.

A strong relationship between the concentration of immigrant students in a school and the school's average familiarity with mathematics concepts is also observed in Estonia, Liechtenstein,

■ Figure 2.7 ■

Percentage of immigrants in schools, by school-level familiarity with mathematics



Notes: Schools where students are less (more) familiar with mathematics are defined as those where the students' average level on the *index of familiarity with mathematics* is statistically significantly below (above) the average across all schools in the country/economy.

The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).

Countries and economies are ranked in ascending order of the percentage of students with an immigrant background in all schools.

Source: OECD, PISA 2012 Database, Table 2.6.

StatLink http://dx.doi.org/10.1787/888933377049

Luxembourg and Switzerland. In these countries, the difference in the percentage of immigrant students between schools with less and more familiarity with mathematics is larger than 15 percentage points. These differences reflect both the level of skills among immigrant students in their respective host countries and the concentration of students with knowledge deficits who attend the same school (Figure 2.7).

Teachers and school administrators face the challenge of teaching increasingly multiethnic and multilingual classes. Many of them recognise that handling cultural diversity in class is difficult and requires preparation. On average across the 21 OECD countries that participated in the OECD Teaching and Learning International Survey (TALIS) in 2013,[4] 12% of teachers reported they needed professional development for teaching in multicultural settings (OECD, 2015b). This feeling of unpreparedness was shared by 27% of teachers in Italy and 33% of teachers in Mexico.

Many believe that mathematics is a subject free of the influence of culture, beliefs and values, and it can be taught even in the absence of a common language, because it is, in itself, a universal language. In reality, cultural beliefs about mathematics affect teaching practices and influence immigrant students' participation in the classroom and learning (Gorgorió and Planas, 2005). Immigrant students might differ not only in their background knowledge, but also in the strategies they use to solve problems. For example, mathematics teachers can choose among many different representations of the algorithm of division, and this choice is often culture-specific. Teachers who are not fully aware of these differences in approaches to mathematics problems or who "play down" cultural differences, arguing for general notions of ability and equity (Abreu, 2005), are ill-equipped to build on their students' knowledge and experience with mathematics.

On average across OECD countries, only 4% of students are in schools whose principal reported that ethnic heterogeneity is a serious obstacle to learning (Table 2.8). Not surprisingly, principals of socio-economically disadvantaged schools (that is, schools where the average socio-economic status of students is statistically significantly below that of the average school in the country/economy) are much more likely than principals of advantaged schools to report that ethnic diversity hinders learning very much. This is particularly the case in Belgium, where ethnic heterogeneity is perceived as a serious obstacle to learning by principals in 20% of disadvantaged schools. These perceptions reflect the fact that immigrant students, who have, arguably, the largest learning and linguistic deficits, tend to be concentrated in disadvantaged schools. They also make it clear that disadvantaged schools need more support so they can start regarding ethnic differences as a resource for learning, rather than an obstacle to learning (OECD, 2015b).

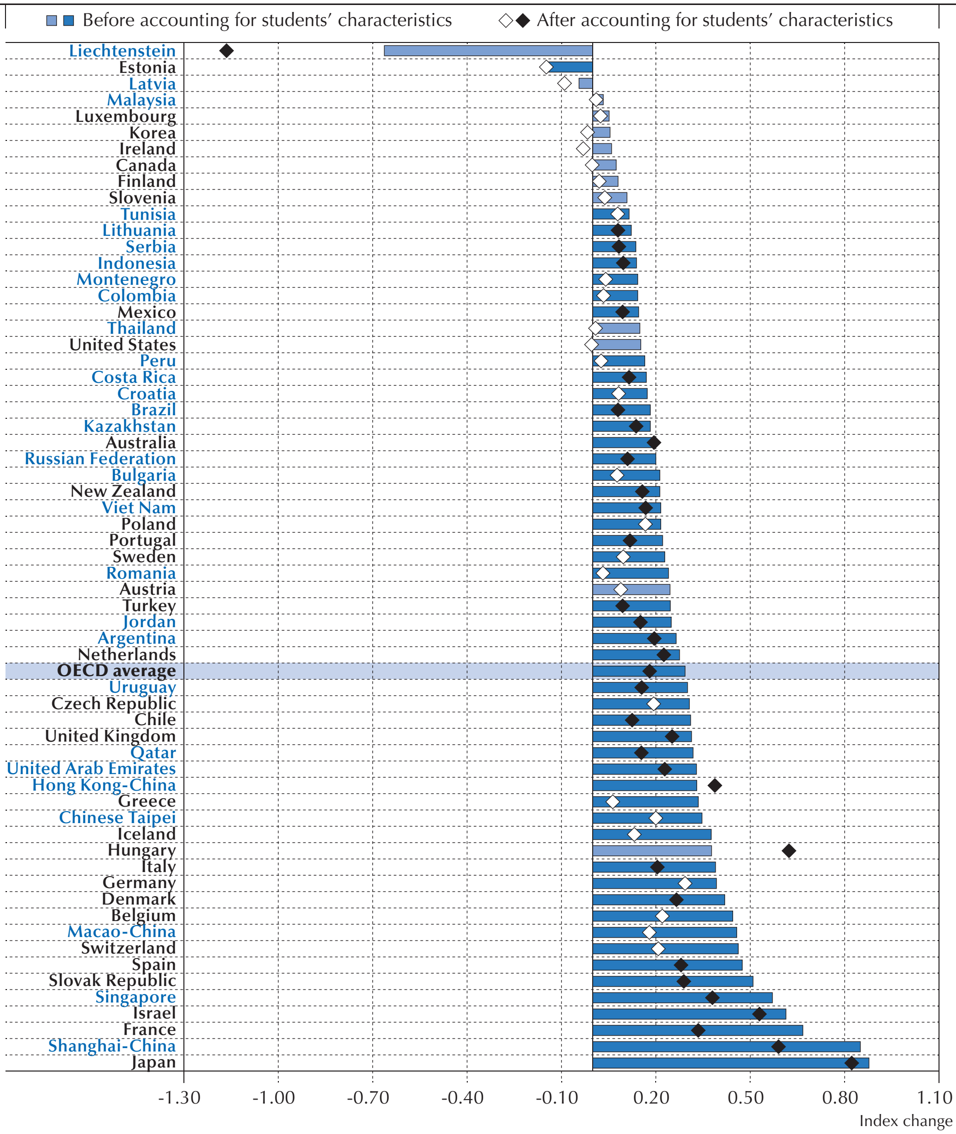
Attendance at pre-primary education and familiarity with mathematics

Very young children have the potential to learn mathematics that is complex and sophisticated (Sarama and Clements, 2010), and pre-primary education can help children with their first steps towards mathematical literacy. Identifying the relationship between pre-primary education and later performance in school is challenging, because attendance at pre-school is often correlated with socio-economic advantage. When disadvantaged children enter pre-school, they already lag behind advantaged children because they are likely to have had fewer play opportunities at home to explore patterns, shapes and spatial relations; compare magnitudes; and count objects.

■ Figure 2.8 ■

Familiarity with mathematics and attendance at pre-primary education

Change in the index of familiarity with mathematics associated with attendance at pre-primary education



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).
"Students' characteristics" include students' gender, socio-economic status, immigrant background and language spoken at home.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in ascending order of the change in the index of familiarity with mathematics *associated with attendance at pre-primary education before accounting for students' characteristics.*
Source: OECD, PISA 2012 Database, Table 2.9b.
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A growing body of experimental studies have overcome this analytical challenge and shown that policy interventions in early childhood education can yield large returns in general, and be particularly effective for children in low-income families (Heckman and Carneiro, 2003; Blau and Currie, 2006; Cunha et al., 2006).

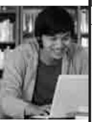
At age 15, students who had attended pre-primary education report greater familiarity with mathematics than those who had not attended (Figure 2.8). The knowledge advantage of students who had attended pre-primary education remains substantial (around one-fifth of a standard deviation, on average across OECD countries) even after accounting for other student characteristics, such as their gender, socio-economic status, and immigrant background. Pre-primary education makes a large difference in those countries, like Hungary, where attendance at these programmes is almost universal. On average across OECD countries in 2012, 89% of disadvantaged students and 96% of advantaged students had attended at least one year of pre-primary education (Table 2.9a).

Unfortunately, many disadvantaged children attend pre-schools that are not of high quality. For example, evidence from the United States shows that children from poor neighbourhoods are more likely than children from wealthier communities to be taught by teachers with fewer qualifications (Clifford et al., 2005; Sarama and Clements, 2010). The approaches used for teaching mathematics in pre-primary schools might make a difference for building a sturdy base for further learning. In pre-primary mathematics activities, the content is usually not the main focus, but is embedded in a fine-motor or reading activity. Experimental evidence suggests that a lack of explicit attention to mathematics concepts and procedures, along with a lack of intention to engage in mathematics practice, results in insufficient opportunities to build strong cognitive foundations (Clements and Sarama, 2011). The same evidence indicates that interventions that provide early experience with numbers, space, geometry, measurement, and the processes of mathematical thinking can be particularly effective for children from poorer communities.

PARENTS' PREFERENCES, SCHOOL SELECTIVITY AND OPPORTUNITY TO LEARN MATHEMATICS

In recent decades, reforms in many countries have tended to give a greater say in school choice to parents and students (Heyneman, 2009). Parents' background and preferences are important for school choice, especially in school systems with early tracking and where school selection is not based on achievement. Parents' criteria for choosing schools are a component of the effect of socio-economic status on opportunity to learn, as wealthier parents tend to have access to the information needed to select the best schools for their children.

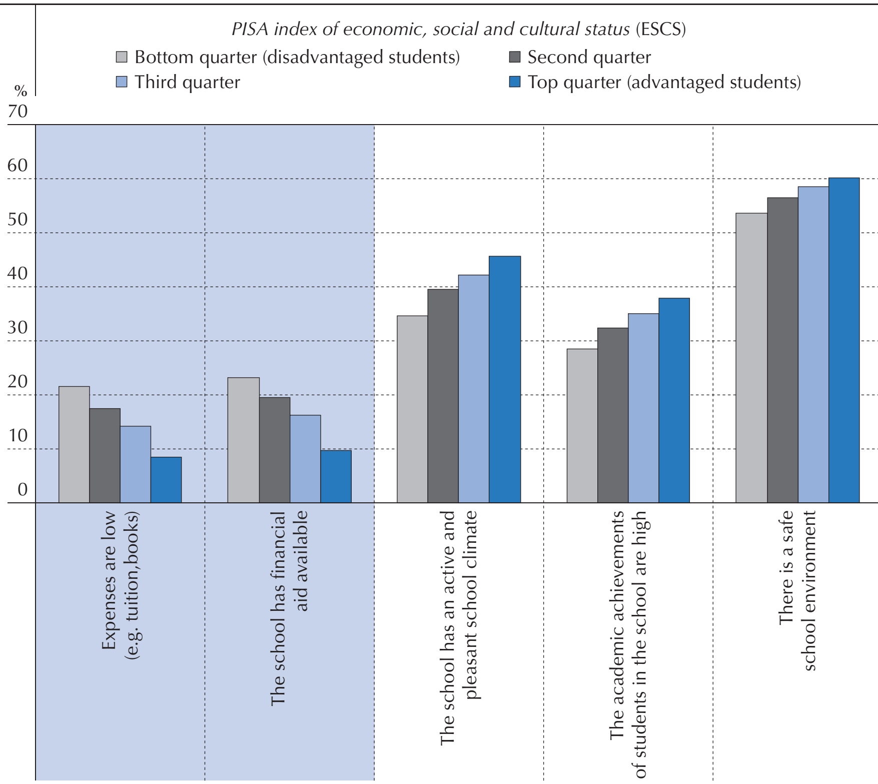
Even if all parents would like their child to attend the best schools, not everyone can afford to care only about school quality when choosing a school. Figure 2.9 shows how, across the 11 countries and economies where the parent questionnaire was distributed, disadvantaged parents tended to assign relatively greater importance to financial considerations when choosing a school for their child.



■ Figure 2.9 ■

Parents' preferences for schools, by socio-economic status

Percentage of students whose parents reported that the following criteria are "very important" in choosing a school for their child; average across 11 countries/economies



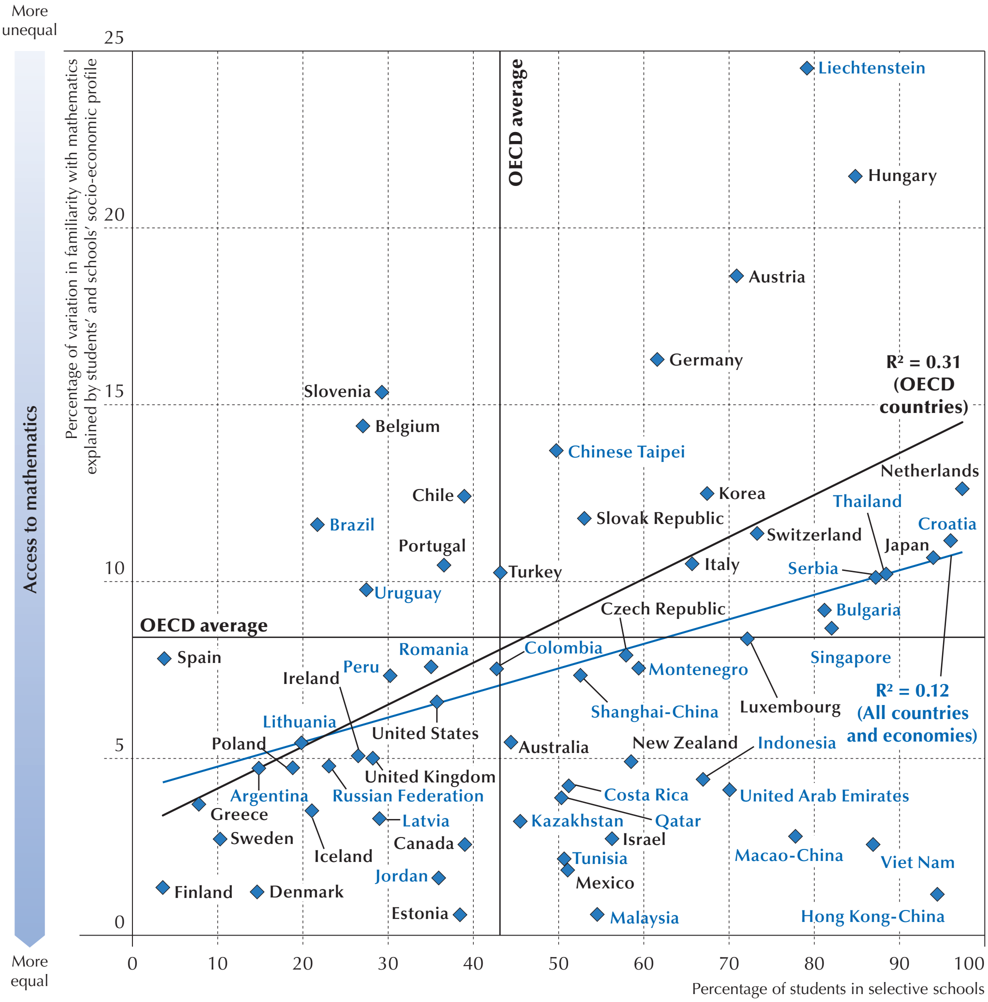
Notes: Parents' reports on their criteria for choosing schools for their children, by students' socio-economic status.
Only the following countries and economies with available data from the parent questionnaire are shown: Belgium (Flemish community), Chile, Croatia, Germany, Hong Kong-China, Hungary, Italy, Korea, Macao-China, Mexico and Portugal.
Source: OECD, PISA 2012 Database, Table 2.11.
StatLink *http://dx.doi.org/10.1787/888933377063*

Schools' practices of selecting students by academic achievement have a similar effect of reinforcing inequalities. Schools that select students for admittance based on their academic performance tend to show better average performance; but in systems with more academically selective schools, the impact of students' and schools' socio-economic profile on student performance is stronger (OECD, 2013b). Moreover, selective education systems are also linked with greater inequality in social outcomes later in life (Burgess, Dickson and Macmillan, 2014).

School selectivity is also associated with more unequal opportunities to learn mathematics. As shown in Figure 2.10, in Croatia, Hong Kong-China, Japan and the Netherlands, over 90% of students attend selective schools, i.e. schools where student academic performance and/or recommendations from feeder schools are always considered for admission.

■ Figure 2.10 ■

Academic selectivity and equity in familiarity with mathematics



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Selective schools are defined as those schools where student academic performance and/or recommendations of feeder schools are always considered for admission.
The vertical axis reports the percentage of variation in the *index of familiarity with mathematics* explained by students' and schools' socio-economic profile. A higher percentage indicates a stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.12.
StatLink *http://dx.doi.org/10.1787/888933377079*

Across OECD countries, 31% of the variation in the association between students' socio-economic status and their familiarity with mathematics concepts is explained by the percentage of students in selective schools. (Across all participating countries and economies, 12% of the variation is so explained).

Admission requirements based on residence make school choice less dependent on families' socio-economic status, particularly if residential segregation is not pervasive. Figure 2.11 shows that over 70% of students in Greece, Poland and the United States attend schools that always consider residence for admission. Across OECD countries, a higher percentage of students in schools that always consider residence for admission is related to a weaker impact of socio-economic status on familiarity with mathematics. Schools' admission requirements account for around 28% of the between-country differences in equity in access to mathematics (defined as the within-country variation in the *index of familiarity with mathematics* explained by the students' and school's socio-economic profile). Again, the relationship between admission requirements based on residence and equity in opportunities to learn mathematics is weaker when looking at all countries and economies, possibly because, in partner countries and economies, people with similar backgrounds tend to live in the same areas to a greater extent than do people in OECD countries.

EQUITY IN OPPORTUNITY TO LEARN AND SORTING STUDENTS

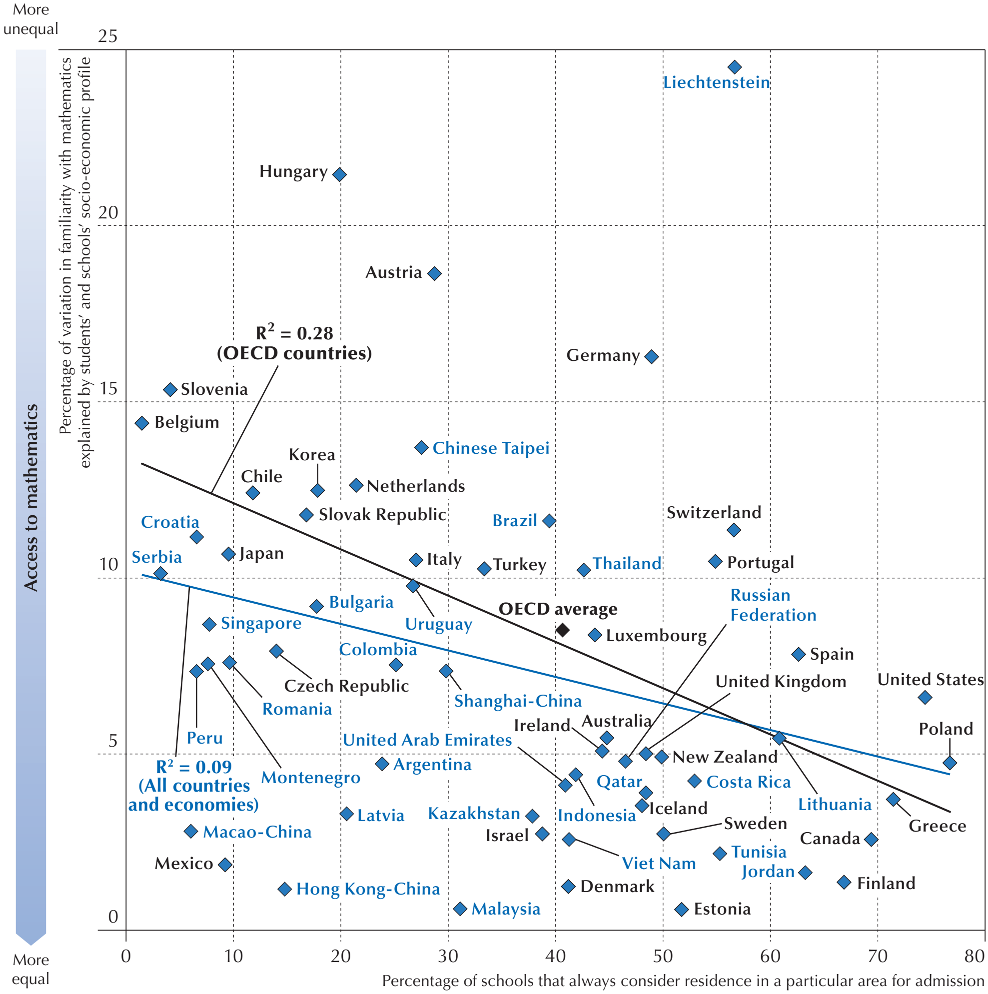
Most school systems aim to improve the effectiveness of teaching by sorting students into relatively homogeneous groups according to their level of achievement. PISA gathers information on how schools and school systems group and select students, known as vertical and horizontal stratification (Figure 2.12). Vertical stratification describes the ways in which students progress through the school system. It is affected by policies governing the age at entry into the school system and grade repetition. Horizontal stratification refers to differences in instruction within a grade or education level, between or within schools, according to students' interests and performance.

Vertical and horizontal stratification are two sides of the same coin: they create opportunities to choose which type of education should be provided to which students. These decisions are based on various factors, not purely on students' abilities. When students are young, they are still in the process of developing their academic potential. These choices are also driven by subjective beliefs: the beliefs that students and their parents hold about their education needs, and the beliefs that teachers, school administrators and regulators hold about the costs of mixing students with different abilities and levels of preparedness in the same classroom. As students navigate this complex system, they face points at which decisions must be made – actively or passively, by the students, their parents or the school – about the next move, with each move affecting the subsequent one (Crosnoe and Schneider, 2010; Morgan, 2005).

In those school systems that sort students into different types of secondary schools or tracks (e.g. vocational or academic), a student's socio-economic status tends to have a strong impact on which school or track is selected. Other systems may have fewer of these distinct "branching points" of sorting, but differences in opportunity to learn related to socio-economic status are observed nonetheless. For example, disadvantaged students tend to select less academically challenging mathematics courses, especially when those courses are elective (Csikszentmihalyi and Schneider, 2000).

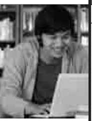
■ Figure 2.11 ■

Residency requirements and equity in familiarity with mathematics



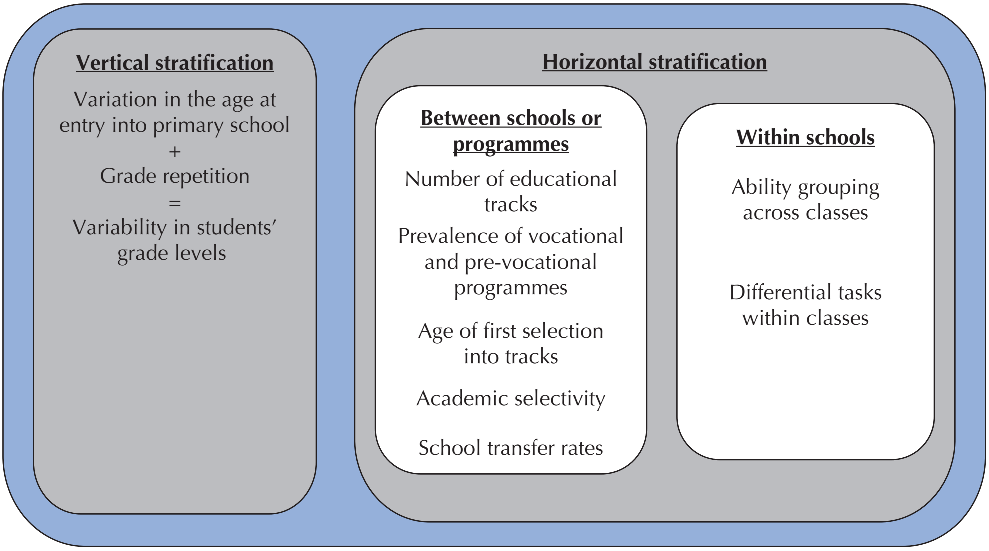
Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The vertical axis reports the percentage of variation in the *index of familiarity with mathematics* explained by students' and schools' socio-economic profile. A higher percentage indicates a stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.13.
StatLink *http://dx.doi.org/10.1787/888933377086*

The cumulative effect of socio-economic status on access to mathematics content throughout a student's career can be easily illustrated through PISA data. PISA assesses students who are between the ages of 15 years 3 months and 16 years 2 months. In several education systems, it is thus possible to observe both students who are immediately before, and students who are



■ Figure 2.12 ■

Selecting and grouping students



Source: OECD (2013).

immediately after, one key branching point: the transition from lower secondary education (ISCED 2) to upper secondary education (ISCED 3).

As students progress from lower to upper secondary education, their familiarity with mathematics becomes more dispersed and more correlated with their socio-economic status (Table 2.14). Figure 2.13 shows that the impact of students' socio-economic status on their familiarity with mathematics concepts tends to be stronger among students in the first year of an upper secondary programme than among students in the last year of a lower secondary programme, after taking into account whether they had repeated a grade at least once in primary or secondary school.[5] This result confirms that the more choices and "permanent" transitions are allowed in a school system, the stronger the impact of socio-economic status on opportunity to learn.

The solution to this dilemma cannot be a fully inflexible system, with no scope for horizontal shifts; this would limit incentives to excel at school and dramatically constrain the right to express preferences. Rather, education systems can weaken the link between socio-economic status and opportunity to learn by becoming more flexible (granting real opportunities to change tracks and courses) and more objective (making track and course placement more dependent on achievement and students' interests rather than on parents' preferences and background).

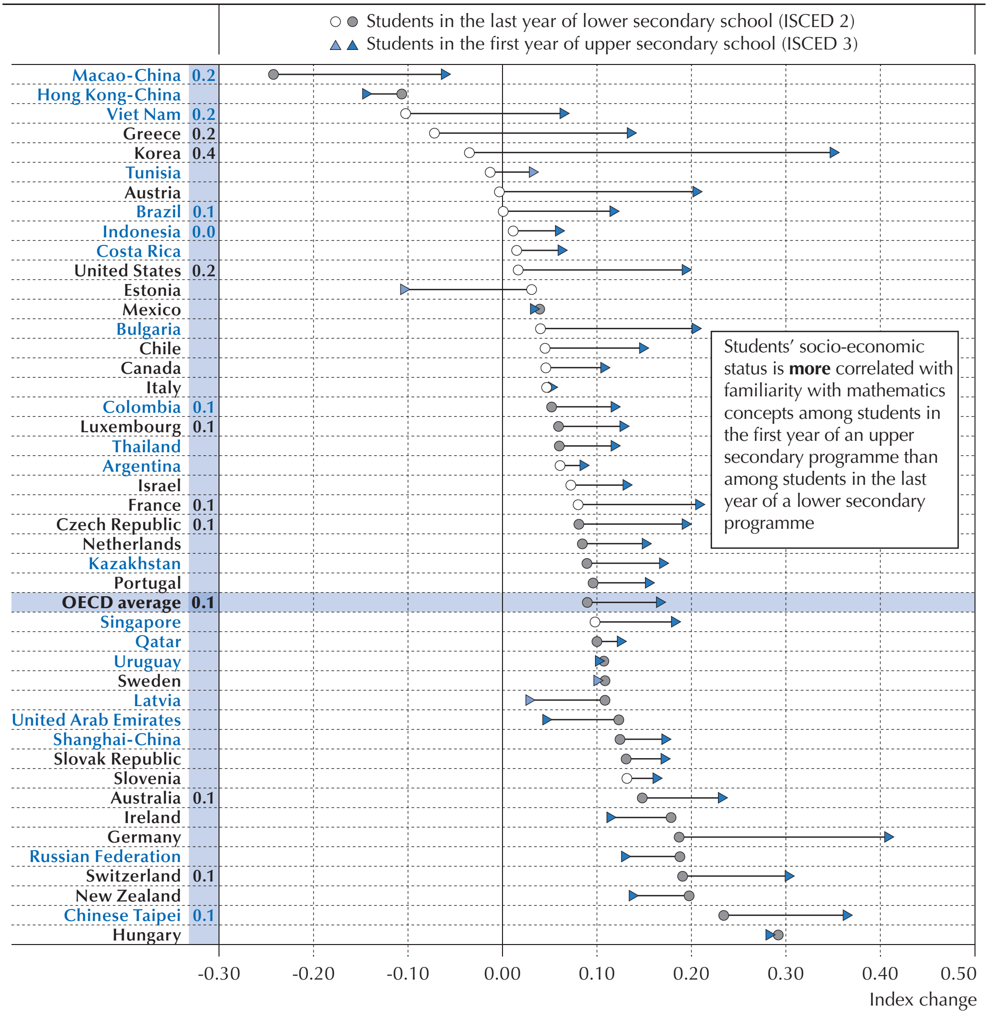
Vertical stratification through grade repetition

Grade repetition, a form of vertical stratification, is used in many systems to give low-performing students a second chance to master their coursework. On average across OECD countries,

■ Figure 2.13 ■

Familiarity with mathematics and students' socio-economic status, by level of education

Change in the index of familiarity with mathematics associated with a one-unit change in the PISA index of economic, social and cultural status (ESCS)



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The analysis takes into account grade repetition (i.e. whether students reported that they had repeated a grade at least once in primary, lower secondary or upper secondary school).
Statistically significant values are marked in a darker tone.
Only statistically significant index change differences between students in upper secondary school (ISCED 3) and students in lower secondary school (ISCED 2) are shown next to the country/economy name.
Countries with available data are shown.
Countries and economies are ranked in ascending order of the index change of students in the last year of lower secondary school (ISCED 2).
Source: OECD, PISA 2012 Database, Table 2.14.
StatLink *http://dx.doi.org/10.1787/888933377092*

13% of students reported that they had repeated a grade at least once in primary, lower secondary or upper secondary school (OECD, 2013c: Figure IV.2.2). Grade repetition might be used not so much to help students who are lagging behind, but rather as a stigmatising, and possibly discriminatory, form of punishment for inappropriate behaviour in class (National Research Council, 1999).

Many studies have shown that grade repetition is not necessarily beneficial for students. In fact, it may increase the probability that students drop out, stay longer in the school system, or spend less time in the labour force (Allen et al., 2009; Alexander, Entwisle and Dauber, 2003; Ikeda and García, 2014; Jacob and Lefgren, 2009; Manacorda, 2010). It is, moreover, costly to education systems, because of the expense of providing an additional year of education for a student, and to the wider society, as it delays a student's entry into the labour market (OECD, 2011).

Previous PISA analysis has shown that grade repetition is negatively related to equity in education. Systems where more students repeat a grade tend to show a stronger impact of students' socio-economic status on their performance (OECD, 2013c: Figure IV.1.4). At the same time, retention rates depend significantly on socio-economic factors (Corman, 2003). On average across OECD countries, socio-economically disadvantaged students are 1.5 times more likely to have repeated a grade than advantaged students who perform at the same level (OECD, 2013c). Immigrant students are almost twice as likely as students without an immigrant background to have repeated a grade, after accounting for both performance and socio-economic status (OECD, 2015b).

Figure 2.14 shows that, across OECD countries, grade repetition is negatively related to equity in access to mathematics. Around 38% of the variation in the impact of students' socio-economic status on their familiarity with mathematics concepts can be explained by differences in the proportion of students who had repeated a grade during their school career. (Across all PISA-participating countries and economies, the association is weaker).

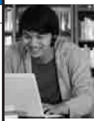
The relationship between grade repetition and equity in opportunities to learn observed across OECD countries does not necessarily imply a causal link, as grade repetition might, in some systems, be a response to, rather than a cause of, differences in students' level of preparedness related to socio-economic inequalities. But given the lack of any solid evidence that repeating a grade improves mastery of mathematics concepts, the economic and social costs of retention are hard to justify.

Horizontal stratification between and within schools and programmes

Various forms of horizontal stratification have been associated with greater inequality in education, as the goal of differentiating curricula by students' achievement level often translates into segregating students by their socio-economic status (Hanushek and Woessmann, 2010; van de Werfhorst and Mijs, 2010; see also Box 2.2).

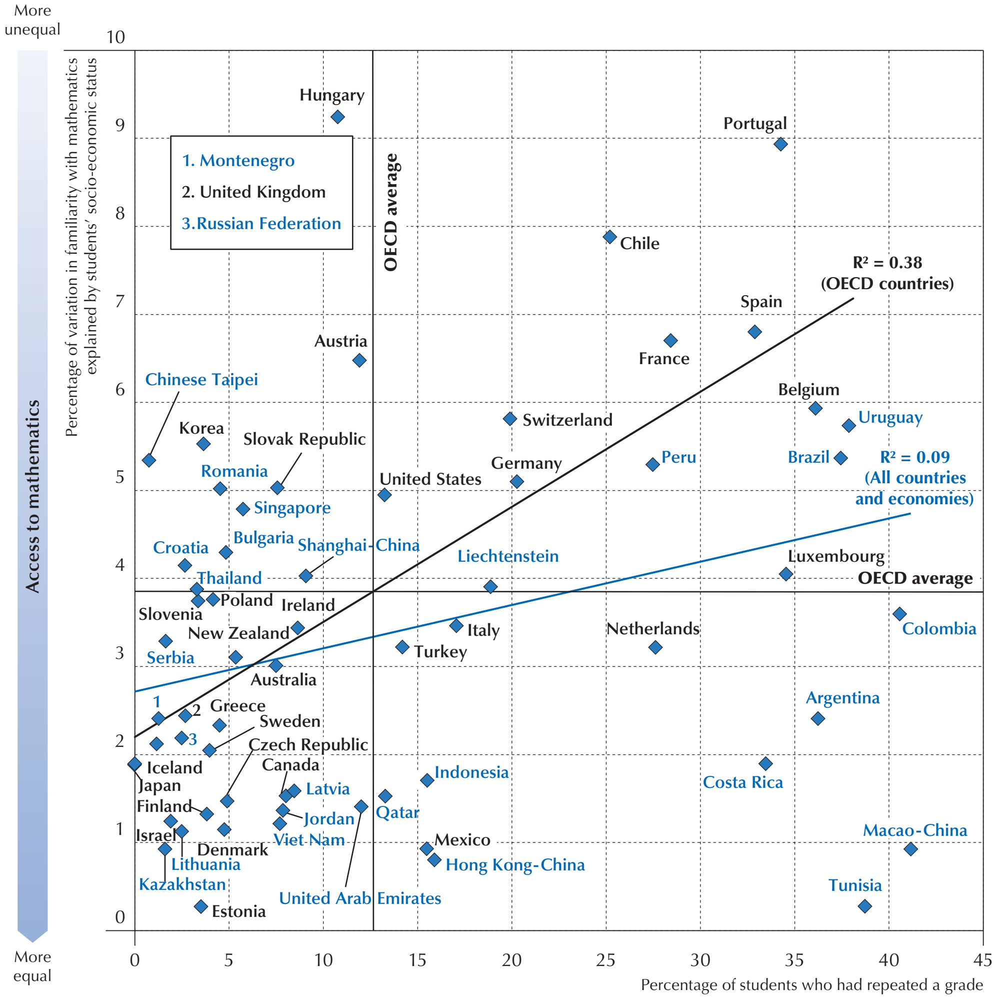
Selection through between-school tracking

Although tracking is widely used – either to sort students into vocational or academic programmes or to base entry into a school on achievement – there is little support for the idea that it positively affects learning (Michaelowa and Bourdon, 2006). In fact, there is considerable international



■ Figure 2.14 ■

Grade repetition and equity in familiarity with mathematics



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The vertical axis reports the percentage of variation in the *index of familiarity with mathematics* explained by the students' socio-economic status. A higher percentage indicates as a stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.15.
StatLink *http://dx.doi.org/10.1787/888933377108*

Box 2.2. Trends in between-school and within-school selection

Both between-school and within-school selection aim to differentiate curricula by students' achievement levels and are widely practiced in OECD countries. In several countries, within-schools ability grouping has increased with the decline or the postponement of between-school tracking.

Since the 1960s a number of developed countries have started reforming their systems by delaying the age of streaming into schools with different orientations or by creating comprehensive schools, including Finland, France, Germany, Norway, Poland, Spain, Sweden, the United Kingdom and the United States (Ariga et al., 2005; Heidenheimer, 1974; Lucas, 1999; Pischke and Manning, 2006; Pekkarinen, Uusitalo and Kerr, 2009). At the same time, ability grouping or other forms of within-school tracking have become more common in some of the same countries, such as France, Germany, the United Kingdom and the United States (Duru-Bellat and Suchaut, 2005; Feinstein and Symons, 1999; Lucas, 1999; Kämmerer, Köller and Trautwein, 2002).

Internationally, countries with the highest rates of course-by-course tracking are Anglophone countries (Australia, Canada, New Zealand, the United Kingdom and the United States), the countries with moderate rates are Nordic and other comprehensive systems (Iceland, Norway, Poland, Spain and Sweden), and the countries with low rates are Denmark and Finland, as well countries practicing primarily academic/vocational streaming (Austria, Germany, Greece and Japan).

Source:

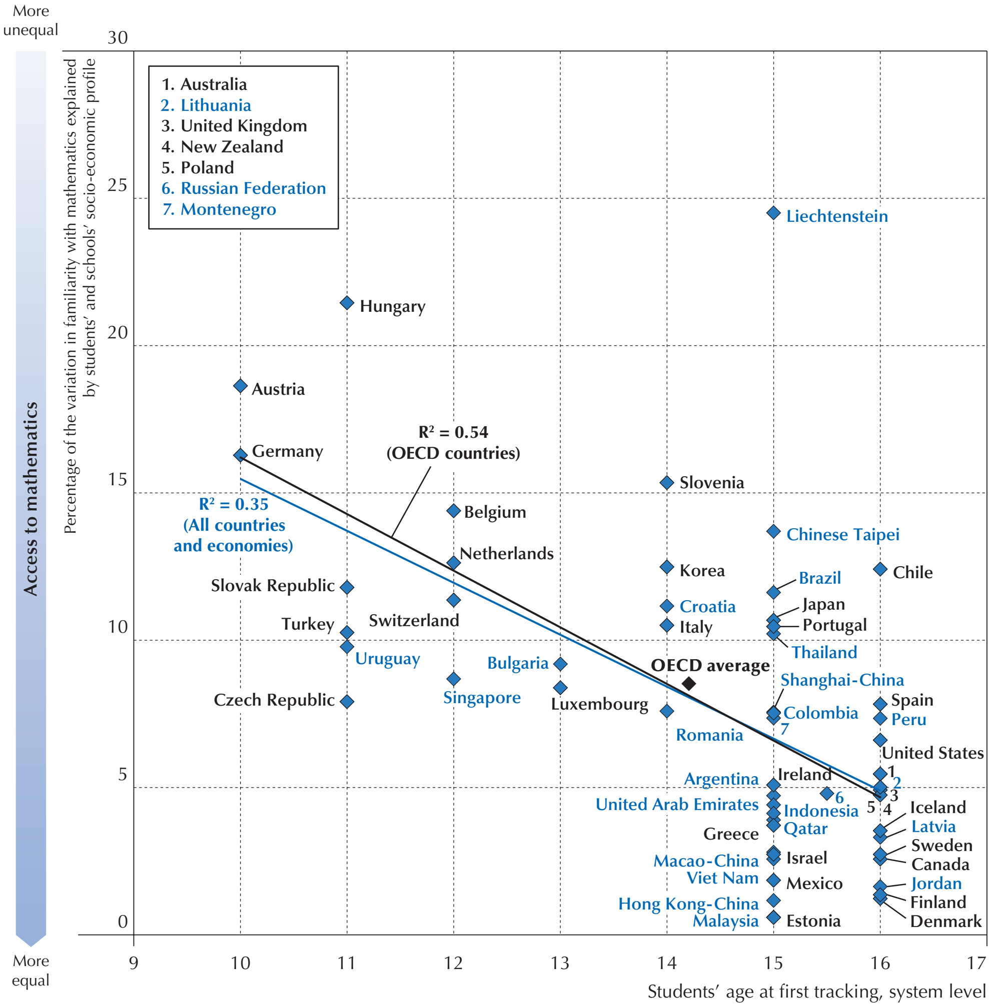
Chmielewski (2014).

evidence that tracking, especially early tracking, is associated with inequality in education, both in student performance (Hanushek and Woessmann, 2006) and in the extent to which individual student achievement and other life outcomes, such as enrolment in tertiary education and earnings in the labour market, reflect family background (Ammermüller, 2005; Brunello and Checchi, 2007; Ferreira and Gignoux, 2014; Horn, 2009; Schütz, Ursprung and Woessmann, 2008; Woessmann et al., 2009). Previous PISA analysis has also shown a negative association between early tracking and equity in education at the system level (OECD, 2013c).

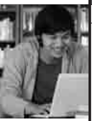
Similarly, PISA 2012 data show that early tracking is related to inequalities in opportunities to learn mathematics. The relationship between the age at which a student is tracked and equity in access to mathematics is strong: on average across OECD countries, 54% of the variation in the impact of students' and schools' socio-economic profile on students' familiarity with mathematics is explained by system-level differences in the age at which students are first sorted into academic or vocational programmes (Figure 2.15). Across all participating countries and economies, the relationship is somewhat weaker, but still 35% of the variation in equity in access to mathematics is explained by differences in the age at which students are first tracked. In countries like Austria and Germany that start tracking students very early, heterogeneity in overall opportunity to learn is also quite large (as measured by the total variation in familiarity with mathematics within the country, Figure 2.1).

■ Figure 2.15 ■

Age at first tracking and equity in familiarity with mathematics



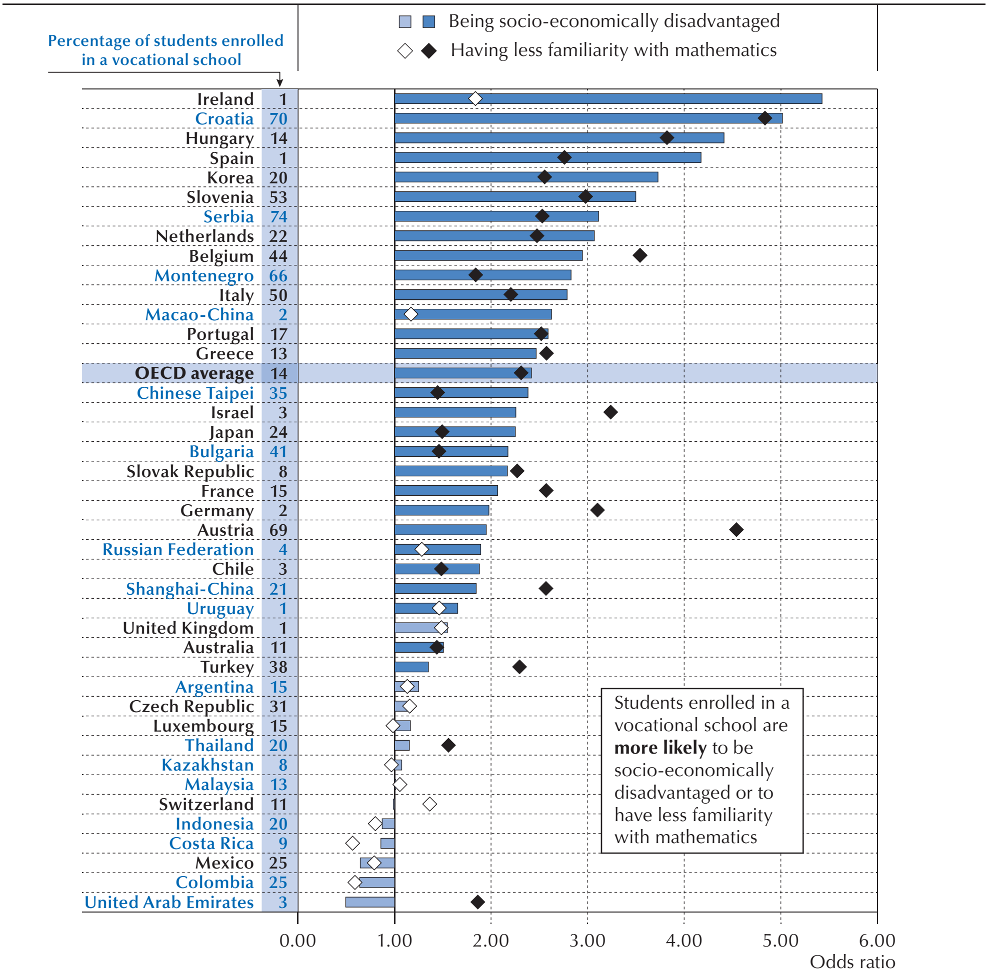
Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The vertical axis reports the percentage of variation in *the index of familiarity with mathematics* explained by students' and schools' socio-economic profile. A higher percentage indicates stronger impact of socio-economic status on students' familiarity with mathematics.
Source: OECD, PISA 2012 Database, Table 2.16.
StatLink *http://dx.doi.org/10.1787/888933377117*



■ Figure 2.16 ■

Concentration in vocational schools of disadvantaged students and students with less familiarity with mathematics

Relationship between enrolment in vocational schools and likelihood of having less familiarity with mathematics and/or being socio-economically disadvantaged



How to read the chart: An odds ratio of 2 for socio-economic status means that a student enrolled in a vocational school is twice as likely to be disadvantaged as a student who is not enrolled in a vocational school. Similarly, an odds ratio of 0.5 for socio-economic status means that a student enrolled in a vocational school is 50% less likely to be disadvantaged than a student who is not enrolled in a vocational school.

Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).
Students with less familiarity with mathematics are students in the bottom quartile of the *index of familiarity with mathematics*.
Disadvantaged students are students in the bottom fourth of the *PISA index of economic, social and cultural status* (ESCS).
Students enrolled in a modular programme are not considered to be enrolled in a vocational school.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in descending order of the odds ratio for disadvantaged socio-economic status.
Source: OECD, PISA 2012 Database, Table 2.17.
StatLink http://dx.doi.org/10.1787/888933377124

The age at which a student is first tracked matters because younger students are more dependent upon their parents, and socio-economically advantaged parents might be in a better position to promote their children's best interests. As students grow older, more information about their abilities is available for educators to make objective evaluations, and for students to know what type of education better suits their preferences and career expectations.

The effect of tracking on equity of opportunity to learn is also related to the share and composition of students attending different types of schools. Almost one in seven students, on average across OECD countries, and more than one in two students in Austria, Croatia, Montenegro, Serbia and Slovenia, attend vocational schools (Table 2.17).

Figure 2.16 shows that, in most countries, students attending a vocational school are disproportionately more likely to be disadvantaged and to have less familiarity with mathematics than students attending an academic school. Students in Croatia, Hungary, Ireland, Korea, the Netherlands, Serbia, Slovenia and Spain who attend vocational schools are more than three times as likely to come from a disadvantaged background as students attending academic schools. Moreover, students in Austria, Belgium, Croatia, Germany, Hungary and Israel who attend a vocational school are over three times as likely to have less familiarity with mathematics as students who attend an academic school.

Ability grouping in different courses within the same school

Grouping students by ability within schools is another way of addressing students' differences in readiness to learn. In several countries, within-school ability grouping has increased with the decline or postponement of between-school tracking (Box 2.2).

PISA 2012 asked school principals to indicate the extent to which differences in students' abilities within classes hinder learning. Figure 2.17 shows that disruption to learning related to differences in students' abilities was reported more frequently by principals in disadvantaged schools than by those in advantaged schools. More than 30% of students in Chile, Croatia, Greece, Thailand and Uruguay attend disadvantaged schools whose principals reported that differences in students' abilities seriously hinder learning.

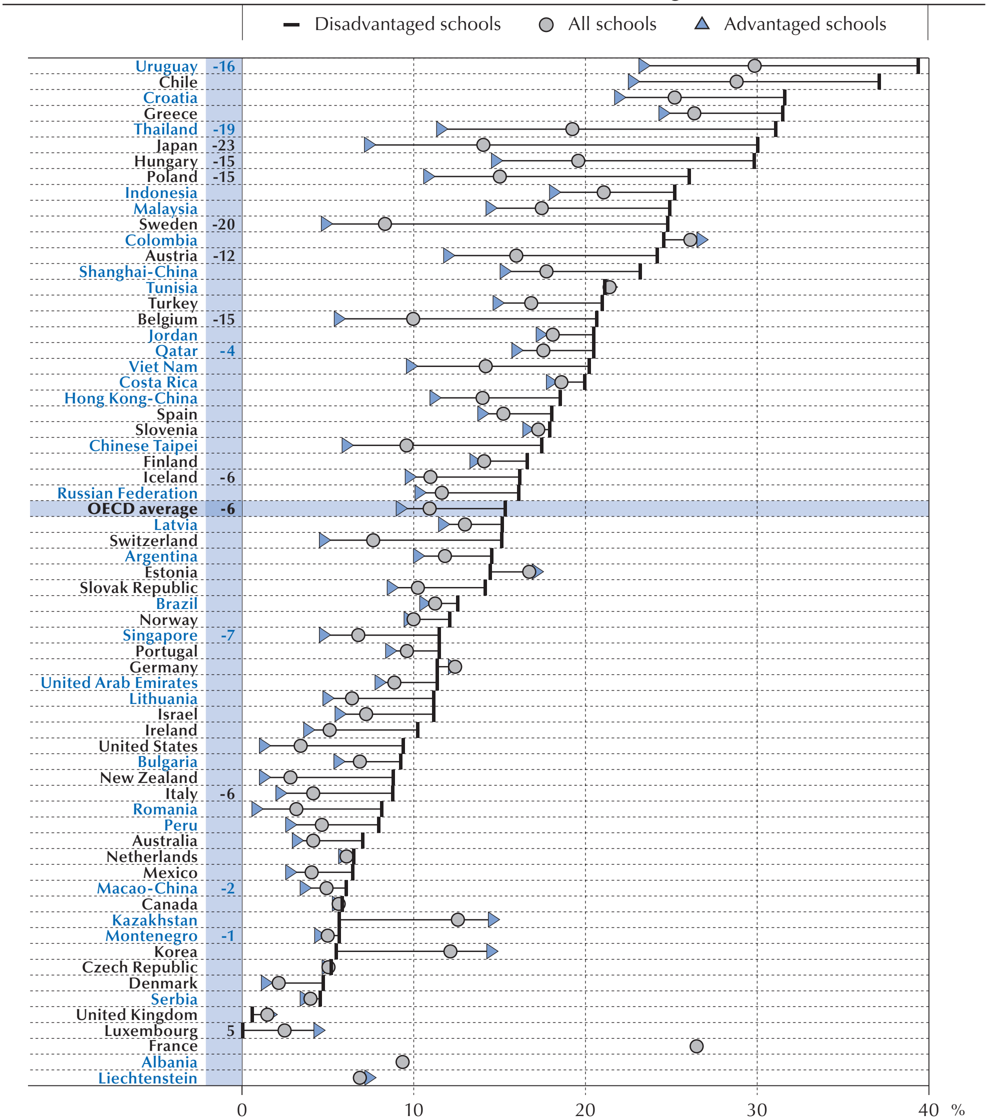
Ability grouping is relatively widespread across OECD countries, with more than 70% of students attending schools whose principal reported that students are grouped by ability for mathematics classes (Figure 2.18a). Over 95% of students in Australia, Ireland, Israel, Kazakhstan, Malaysia, New Zealand, the Russian Federation, Singapore and the United Kingdom attend such schools.

The effect of ability grouping within schools on achievement is unclear. While most studies find positive effects on the performance of high-achieving students, the effects among low-achieving students are open to debate (Argys, Rees and Brewer, 1996; Betts and Shkolnik, 2000; Collins and Gan, 2013; Figlio and Page, 2002; Zimmer, 2003). Moreover, ability grouping seems to reinforce socio-economic inequalities, as does between-school tracking: socio-economically disadvantaged students are disproportionally represented in less-able groups (Braddock and Dawkins, 1993; Oakes, 2005). Indeed, in a study of 20 education systems, Chmielewski (2014) finds that

■ Figure 2.17 ■

Effects of ability differences on the learning environment

Percentage of students in schools whose principal reported that ability differences within classes hinder learning a lot



Notes: Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* (ESCS) is statistically lower (higher) than the mean index across all schools in the country/economy.
Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in descending order of the percentage of students in disadvantaged schools whose principal reported that ability differences within classes hinder learning a lot.
Source: OECD, PISA 2012 Database, Table 2.18.
StatLink http://dx.doi.org/10.1787/888933377131

the relationship between students' socio-economic status and mathematics achievement is even stronger with ability grouping than with tracking between vocational and academic schools.

Data from PISA 2003 show that the type of mathematics classes that students attend as part of their sorting by ability is also related to their socio-economic status. Out of nine countries with available data, socio-economically advantaged students in Australia, Germany, Greece, Hungary, Iceland, Korea, the United Kingdom and the United States were more likely to report that they were attending advanced mathematics classes than disadvantaged students (Table 2.20a). After taking students' mathematics performance into account, advantaged students in Hungary, Korea and the United States were more than 50% more likely to take advanced mathematics than disadvantaged students; advantaged students in Greece and Iceland were more than twice as likely to take advanced mathematics as disadvantaged students after accounting for mathematics performance (Table 2.20b).

Figure 2.18a shows that, on average across OECD countries, ability grouping is more prevalent in disadvantaged than advantaged schools. In Austria, Chile, Croatia, Germany, Iceland, Luxembourg, Mexico, Portugal and Switzerland the difference in the percentage of students attending disadvantaged and advantaged schools that group students by ability is at least 10 percentage points.

Moreover, Figure 2.18b shows that, on average across OECD countries, the practice of grouping students by ability is associated with less familiarity with mathematics. In Austria and Switzerland, students in schools that practice ability grouping in all or some classes are less familiar with mathematics by more than 40% of a standard deviation compared to students in schools that do not group students by ability.

It is unclear, however, whether ability grouping is segregating low-achieving students even further or whether it reflects the schools' level of academic selectivity and is actually used to provide greater assistance to low-performing students in disadvantaged schools. Figure 2.18b also shows that, when comparing students of the same gender and socio-economic status who attend schools with similar socio-economic profiles, the negative association between ability grouping and familiarity becomes weaker or not statistically significant. These results suggest that the relationship between ability grouping and familiarity with mathematics is negative largely because ability grouping is used more often in disadvantaged schools, and because it may be used more as a way to give students who have less familiarity with mathematics more practice, rather than as a way to provide advanced education to gifted students.

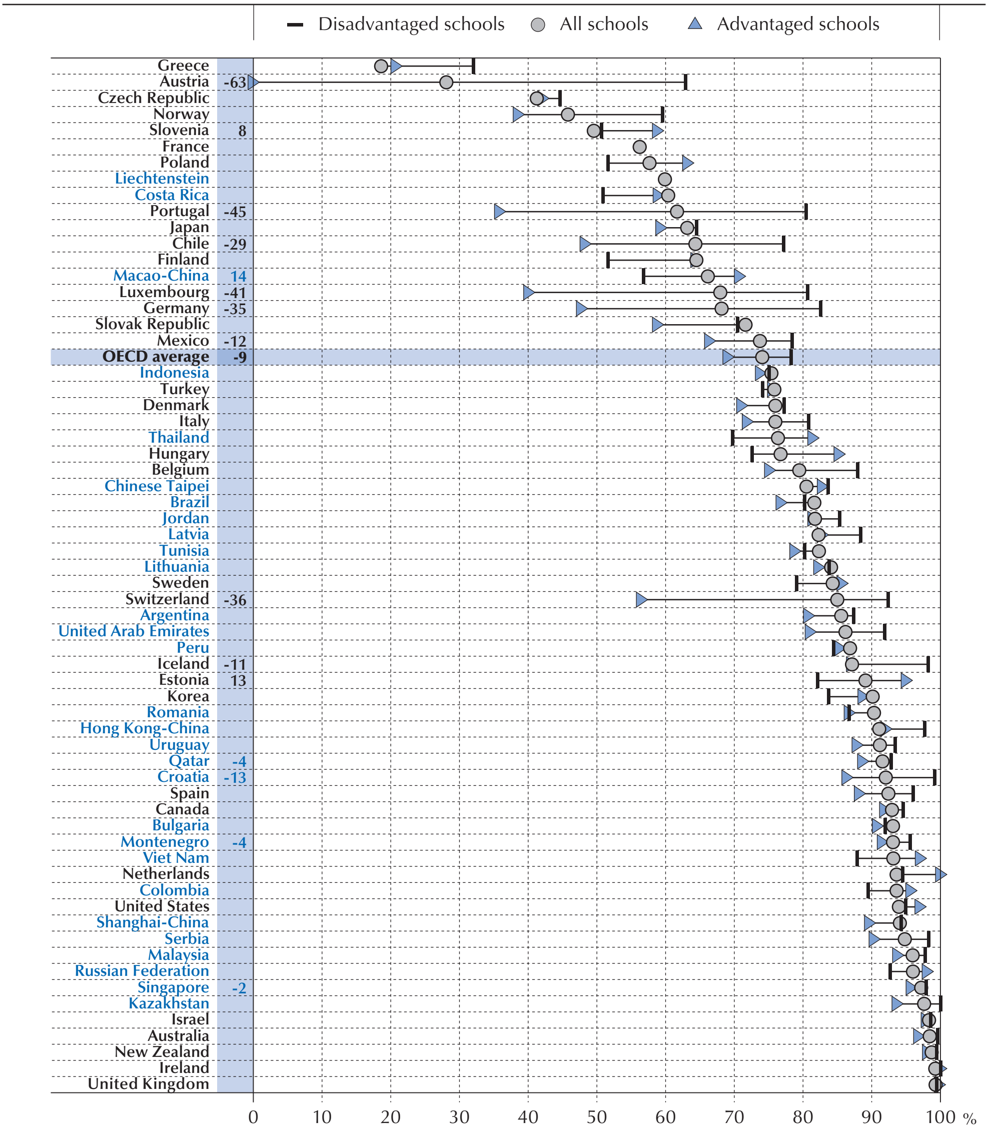
Teaching heterogeneous classes

Teachers are generally committed to providing equal education opportunities; but adapting instruction to each student's skills and needs while advancing learning for all students in the classroom is no small feat. The simplest strategy available to teachers is to use easier mathematics tasks whenever they teach weaker students. According to principals' reports, in most countries and economies at least half of all students attend schools where teachers believe that it is

■ Figure 2.18a ■

Prevalence of ability grouping, by schools' socio-economic profile

Percentage of students in schools whose principal reported that students are grouped by ability for mathematics classes



Notes: Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* (ESCS) is statistically lower (higher) than the mean index across all schools in the country/economy.

Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.

Countries and economies are ranked in ascending order of the percentage of students in all schools.

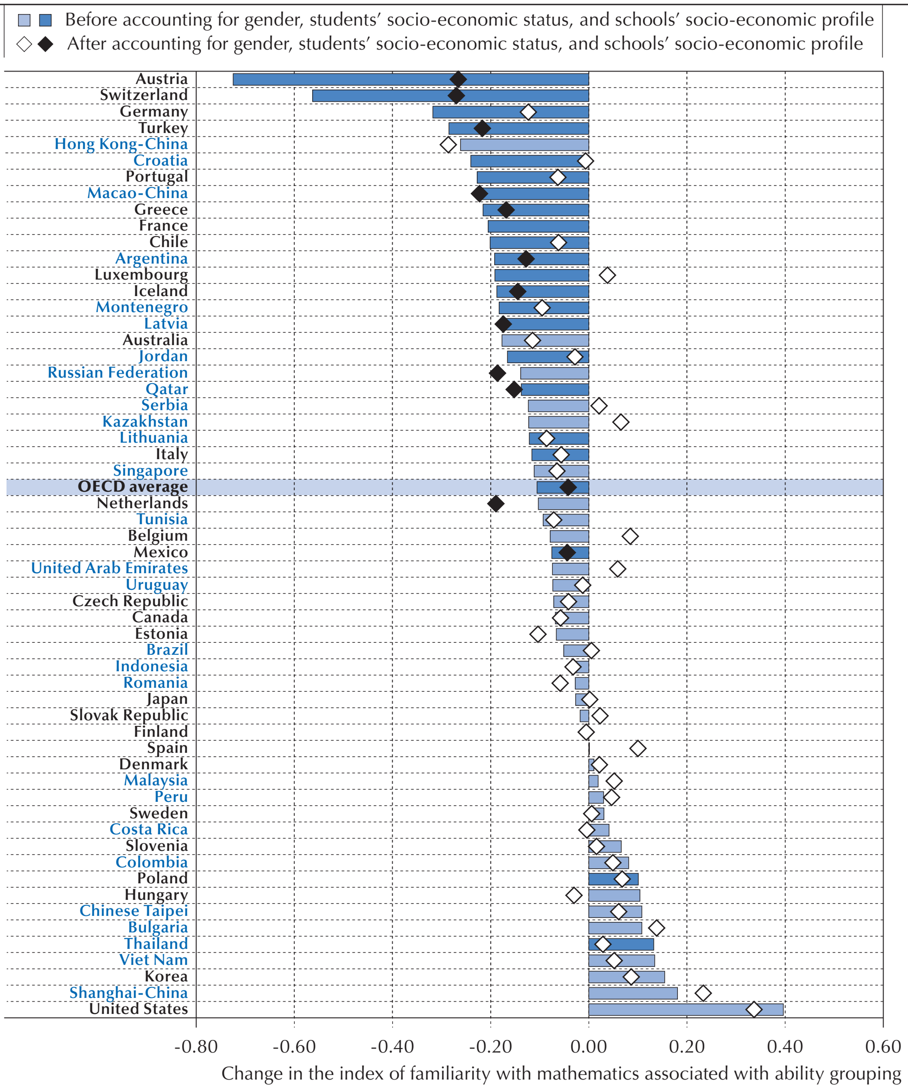
Source: OECD, PISA 2012 Database, Table 2.19a.

StatLink *http://dx.doi.org/10.1787/888933377148*

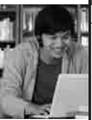
■ Figure 2.18b ■

Ability grouping and students' familiarity with mathematics

Change in students' familiarity with mathematics associated with the school's practice of grouping students by ability in some or all classes



Notes: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (e.g. exponential function, divisor, quadratic function, etc.).
For each student, the school's average familiarity with mathematics is calculated as the average value on the index for all the other students in the school.
Statistically significant values are marked in a darker tone.
Countries and economies are ranked in ascending order of the change in familiarity with mathematics associated with the school's practice of grouping students by ability, before accounting for gender, students' socio-economic status, and schools' socio-economic profile.
Source: OECD PISA 2012 Database, Table 2.19b.
StatLink http://dx.doi.org/10.1787/888933377150



best to adapt academic standards to the students' levels and needs (on average across OECD countries, about 70% of students attend such schools [Figure 2.19]). Differences in teaching cultures and heterogeneity within classes probably explain why teachers in Montenegro, the Russian Federation and most Asian economies are more open to the idea of adapting academic standards than teachers in Austria, Germany and Luxembourg. But teachers' beliefs are also shaped by the environment in which they work, and in the large majority of countries, teachers in disadvantaged schools are much more willing to adjust their academic standards than teachers in advantaged schools.

Another way of teaching students of different abilities within the same class is to assign them different tasks. About 30% of students, on average across OECD countries, reported that teachers in their school differentiate between students when assigning tasks (Figure 2.20). Again, task differentiation is more frequently practiced in disadvantaged than advantaged schools. In Austria, Bulgaria, Germany, the Netherlands, Portugal, Romania, Serbia, the Slovak Republic, Slovenia and the United Arab Emirates, the difference between students in advantaged and disadvantaged schools who reported that their mathematics teacher differentiates tasks according to students' abilities is at least 20 percentage points (Figure 2.20). Assigning different tasks based on students' abilities can better address the needs of low performers, but might, at the same time, prevent low-achieving students from having the same opportunities to learn as higher-achieving students.

Selection through transfers

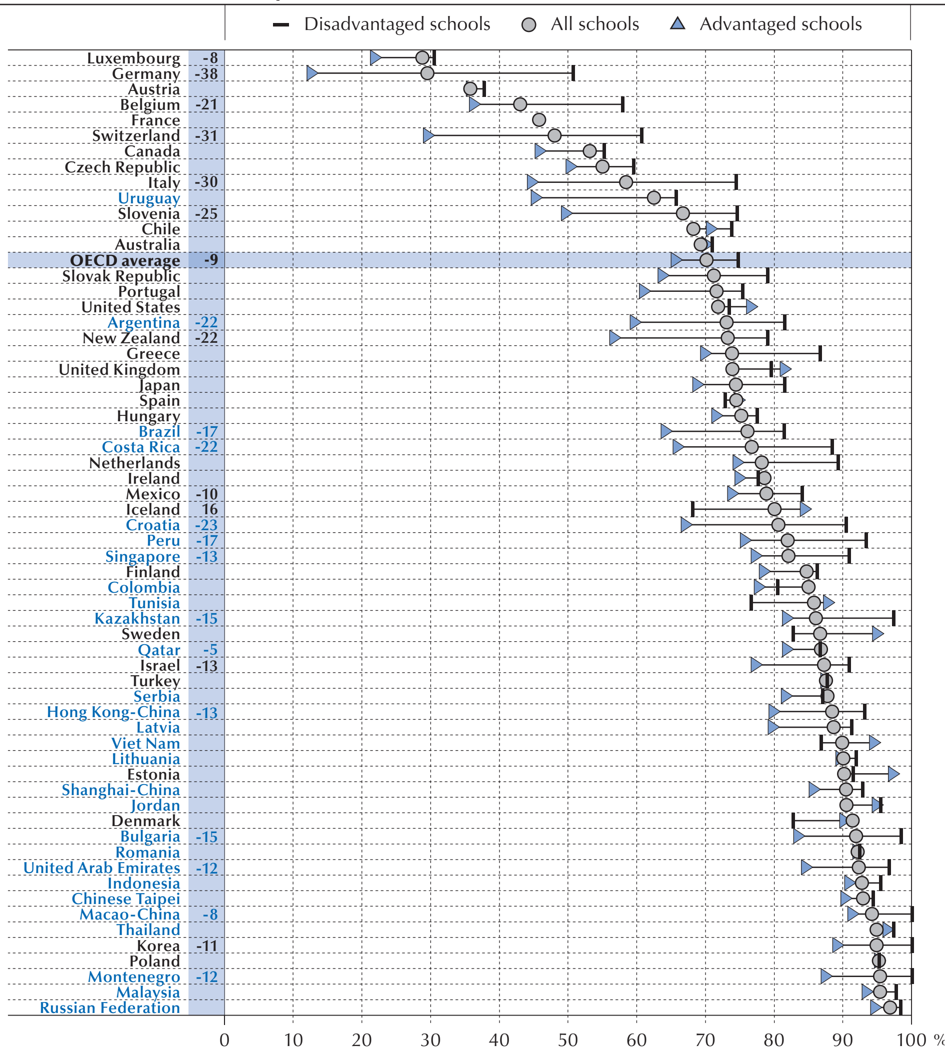
A much more radical way of separating students by ability consists in transferring low-achieving students to other schools (OECD, 2013c: Figure IV.2.6). This highly segregating practice is used more frequently than what might be expected. Over 70% of students in Austria, Macao-China, Slovenia and Chinese Taipei attend schools whose principals reported that they would transfer low-achieving students to another school (Table 2.23). The objectives of this policy might be either to preserve the learning environment of schools already struggling with low performers and poor results on standardised tests, or to assign students with special needs to specially equipped schools.

At the system level, the extent to which students' and schools' socio-economic profile affects familiarity with mathematics is positively related to the practice of transferring low-achieving students. As shown in Figure 2.21, across OECD countries, 42% of the variation in the impact of students' and schools' socio-economic status on students' familiarity with mathematics concepts is explained by the percentage of students in schools that are likely to transfer low-performing students (across all participating countries and economies, 16% of the variation is so explained). This association is fairly easy to explain: young people who are pushed out of a school (or strongly encouraged to leave) are disproportionately poor. The fewer learning opportunities and the social stigma that come from a forced transfer to another school can lead to early dropout and social exclusion (Books, 2010).

■ Figure 2.19 ■

Teachers' beliefs about the need to adapt academic standards to ability

Percentage of students in schools where teachers reported that they believe that it is best to adapt academic standards to the students' levels and needs

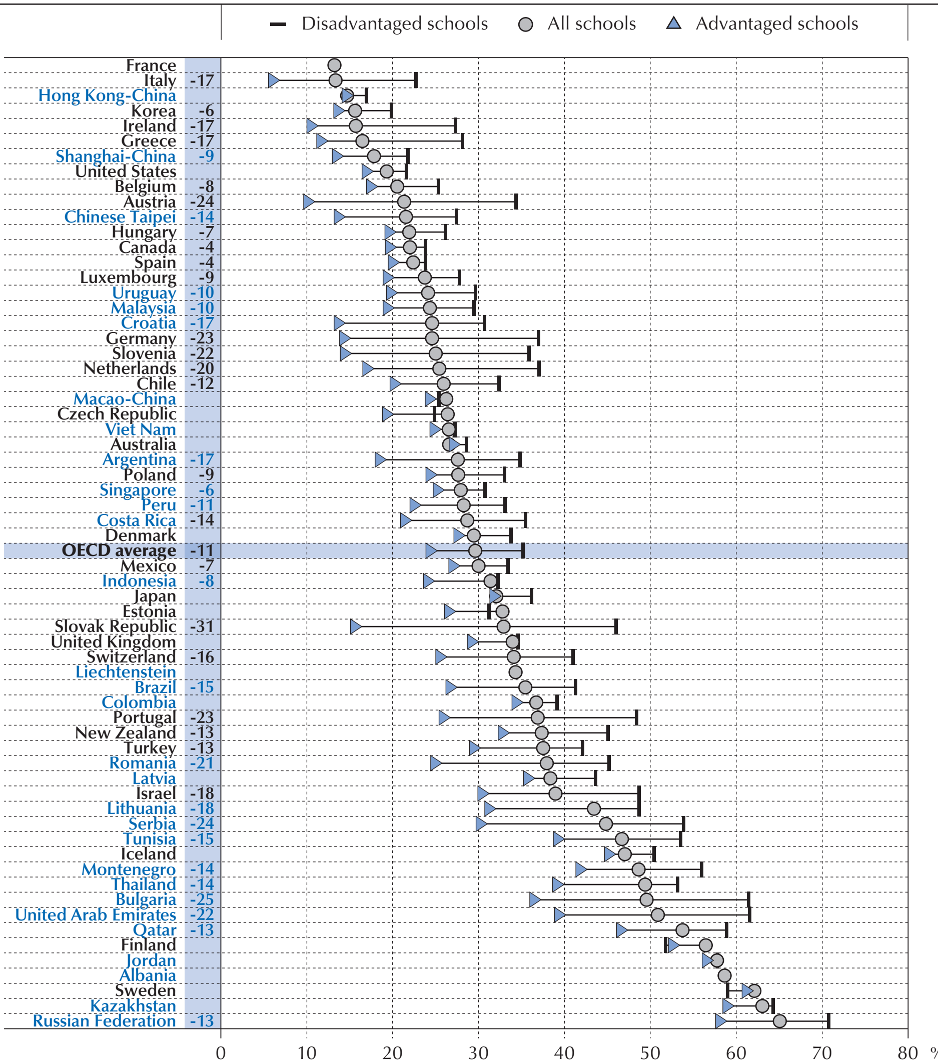


Notes: The figure reports the percentage of students in schools whose principal agreed or strongly agreed that there is consensus among mathematics teachers that it is best to adapt academic standards to students' levels and needs.
Disadvantaged (advantaged) schools are those schools whose mean *PISA index of economic, social and cultural status* (ESCS) is statistically lower (higher) than the mean index across all schools in the country/economy.
Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the percentage of students in all schools whose principal agreed or strongly agreed that there is consensus among mathematics teachers that it is best to adapt academic standards to students' levels and needs.
Source: OECD, PISA 2012 Database, Table 2.21.
StatLink http://dx.doi.org/10.1787/888933377168

■ Figure 2.20 ■

Teachers assigning different tasks to students based on ability, by schools' socio-economic profile

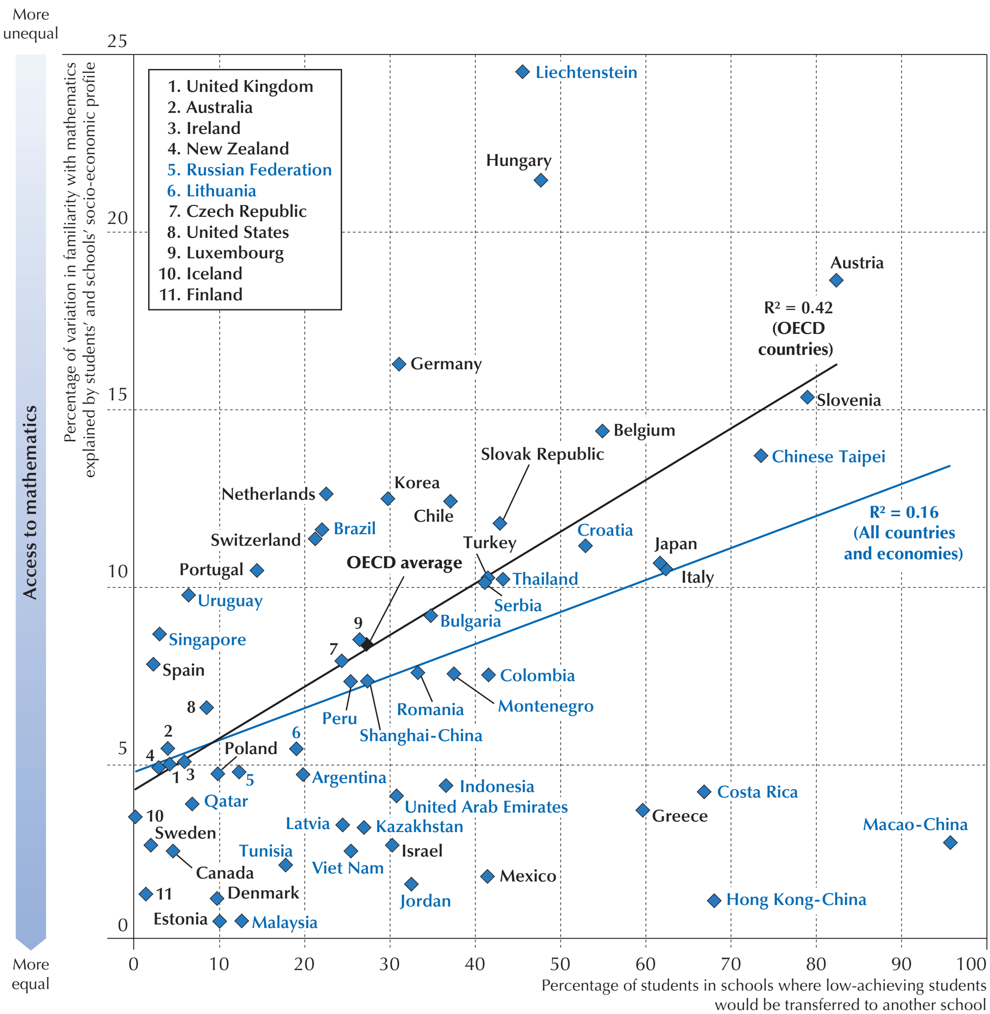
Percentage of students who reported that teachers in their school differentiate between students when assigning tasks



Notes: Task differentiation by teachers is measured on the basis of students' self-reports.
Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.
Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the percentage of students in all schools where teachers differentiate between students when giving tasks.
Source: OECD, PISA 2012 Database, Table 2.22.
StatLink http://dx.doi.org/10.1787/888933377173

■ Figure 2.21 ■

School transfers and equity in familiarity with mathematics



Note: The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
Source: OECD, PISA 2012 Database, Table 2.23.
StatLink http://dx.doi.org/10.1787/888933377186

HOW EQUITY IN OPPORTUNITY TO LEARN IS RELATED TO TEACHING RESOURCES AND PRACTICES

Teachers can influence equity in access to mathematics content not only by grouping students of similar ability and by assigning different tasks to students, based on their ability, but also more directly: through the quantity and quality of the tasks, and by engaging in certain teaching practices. Effective teaching is the most important in-school factor influencing strong academic performance (Chetty, Friedman and Rockoff, 2014; Rivkin, Hanushek and Kain, 2005). Low-achieving students and disadvantaged students stand to gain the most from highly qualified teachers (Gamoran, 1993; Nye, Konstantopoulos and Hedges, 2004), but they are often paired with the least-skilled teachers (Lankford, Loeb and Wyckoff, 2002).

Historically, schools serving poor communities face staffing problems and high rates of teacher turnover. Some teachers might quit because they prefer to work with more advantaged students (Hanushek, Kain and Rivkin, 2004), but most teachers in disadvantaged schools leave because of disciplinary problems, weaker collegial relationships, poor leadership, high student turnover, and general safety concerns that are more pervasive in disadvantaged schools (Gregory, Skiba and Noguera, 2010). Students in disadvantaged schools are thus more likely than their peers in wealthier schools to experience inconsistent staffing from one year to the next and to be taught by teachers who are new to their school and, often, new to the profession (Simon and Johnson, 2015).

Figure 2.22a shows that most countries allocate an equal or larger number of teachers per student in disadvantaged schools than in advantaged schools, even though differences tend to be small. On average across OECD countries, there is one additional student per teacher in advantaged schools than in disadvantaged schools. The main exceptions are Brazil and Turkey, where there are about 7-8 more students per teacher in disadvantaged than in advantaged schools.

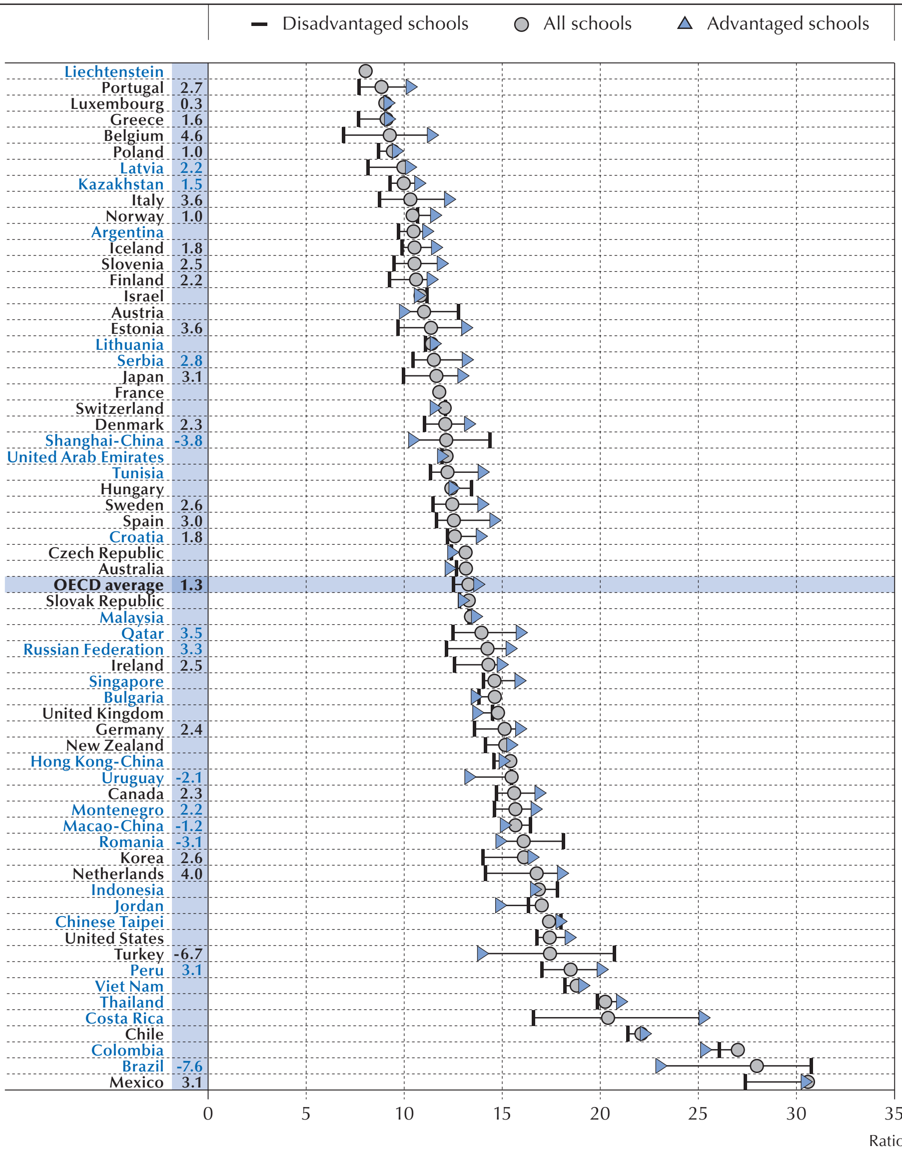
Even though disadvantaged schools have a (slightly) lower student-to-teacher ratio, mathematics teachers in disadvantaged schools tend to be less qualified. Figure 2.22b shows that the percentage of students in schools whose teachers majored in mathematics is generally higher in advantaged schools than in disadvantaged schools. On average across OECD countries, the share of qualified mathematics teachers in advantaged schools is eight percentage points larger than in disadvantaged schools, potentially exacerbating inequalities in opportunities to learn. By contrast, in Finland, Iceland, Macao-China, Spain and the United Arab Emirates, disadvantaged schools have a higher percentage of qualified teachers than advantaged schools.

Teaching practices as well as teachers' qualifications affect students' opportunity to learn. PISA asked students how often their mathematics teachers engage in cognitive-activation strategies, that is instructional practices involving challenging tasks, the activation of prior knowledge and higher-level thinking (Lipowsky et al., 2009). In particular, PISA asked students how often their mathematics teachers adopt the following practices:

- ask questions that make students reflect on the problem
- give problems that require students to think for an extended time

■ Figure 2.22a ■

Number of students per teacher, by schools' socio-economic profile

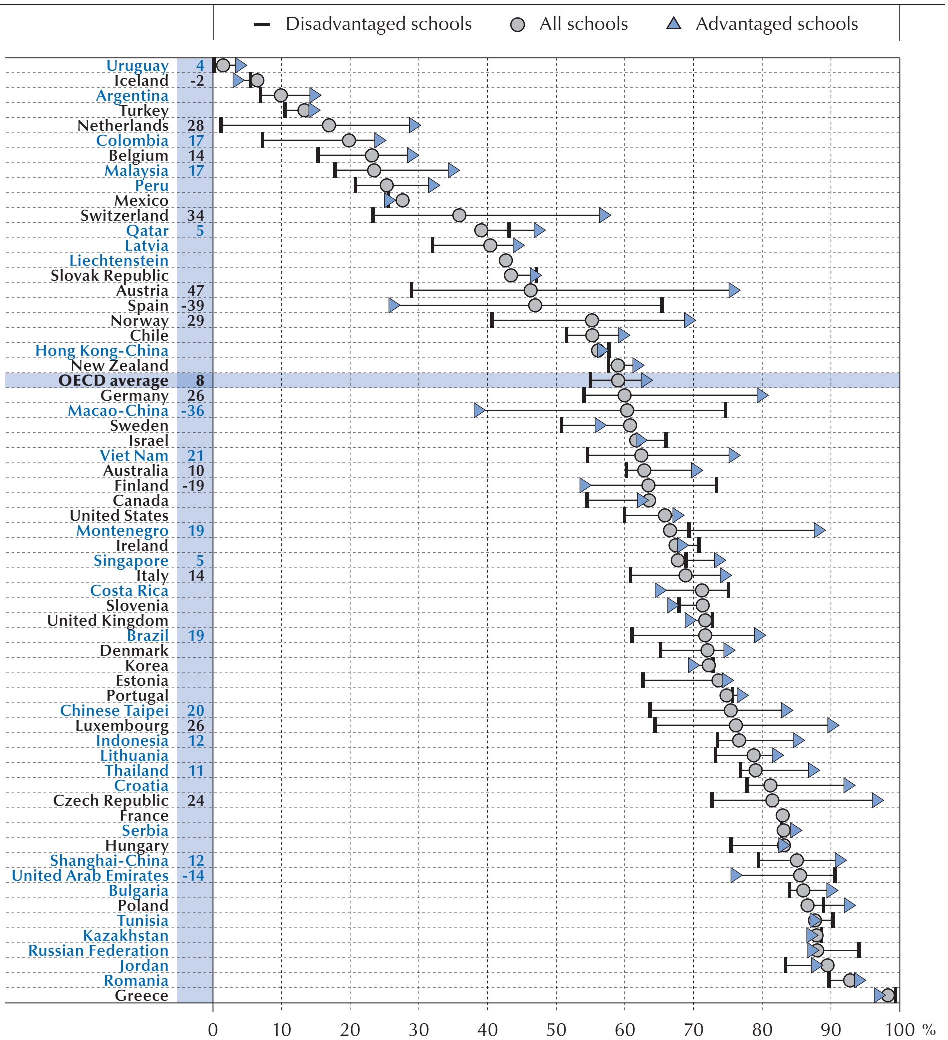


Notes: Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.
Only statistically significant differences in the ratio between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the number of students per teacher in all schools.
Source: OECD, PISA 2012 Database, Table 2.24.
StatLink *http://dx.doi.org/10.1787/888933377197*

■ Figure 2.22b ■

Percentage of qualified mathematics teachers, by schools' socio-economic profile

Percentage of students in schools whose principal reported that mathematics teachers are qualified



Notes: Qualified mathematics teachers are those teachers with a major in mathematics (ISCED 5A). The percentage of qualified mathematics teachers are reported by the school's principal.
Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.
Only statistically significant percentage-point differences between advantaged and disadvantaged schools are shown next to the country/economy name.
Countries and economies are ranked in ascending order of the percentage of qualified mathematics teachers in all schools.
Source: OECD, PISA 2012 Database, Table 2.24.
StatLink http://dx.doi.org/10.1787/888933377200

- ask students to decide on their own procedures for solving complex problems
- present problems for which there is no immediately obvious method of solution
- present problems in different contexts so that students know whether they have understood the concepts
- help students to learn from mistakes they have made
- ask students to explain how they have solved a problem
- present problems that require students to apply what they have learned to new contexts
- assign problems that can be solved in several different ways.

Previous analysis of PISA data shows that students who indicated that their mathematics teacher uses cognitive-activation strategies reported particularly high levels of perseverance and openness to problem solving, were more likely to favour mathematics as a field of study over other subjects, and/or were more likely to regard mathematics as more necessary to their prospective careers than other subjects (OECD, 2013b).

Cognitive-activation strategies tend to be used more often in socio-economically advantaged schools than in disadvantaged schools (Table 2.25a). This is especially the case for strategies that require problem-solving skills and go beyond simple coverage of the curriculum. For instance, on average across OECD countries, the share of students whose teachers assign problems with no obvious solutions is seven percentage points larger in advantaged schools than in disadvantaged schools; and the share of students whose teachers require that they apply what they have learned to new contexts is five percentage points larger in advantaged than in disadvantaged schools. By contrast, "helping students to learn from mistakes they have made" is a strategy reported more often in disadvantaged schools than in advantaged schools, possibly because this strategy is more frequently used to help low-achieving students.

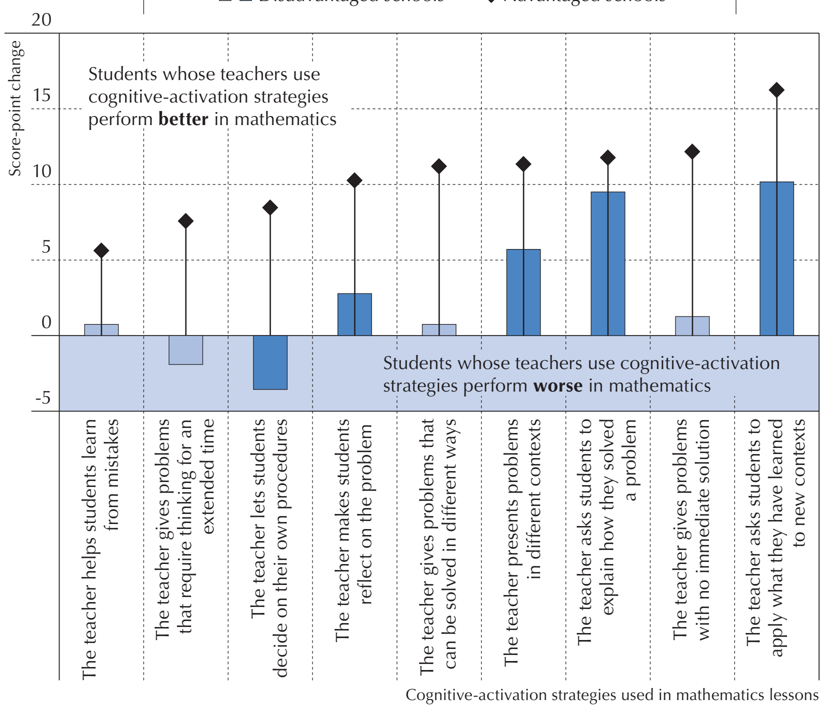
Previous research has shown a positive association between the use of cognitive-activation strategies and mathematics achievement (Echazarra et al., 2016; Lipowsky et al., 2009). What is the relationship between cognitive-activation strategies and opportunity to learn? And how does it vary according to schools' socio-economic profile? Figure 2.23a shows the change in mathematics performance associated with exposure to these strategies, while Figure 2.23b shows the change in familiarity with mathematics associated with the strategies. On average across OECD countries, the use of cognitive-activation strategies is associated with higher scores, in both advantaged and disadvantaged schools. However, in disadvantaged schools, only four out of nine such strategies are associated with better mathematics performance, while all strategies are associated with better performance in advantaged schools. Moreover, the effect on performance is larger in advantaged schools then in disadvantaged schools.[6]

Differences in the effect of cognitive-activation strategies according to schools' socio-economic profile are even more striking when looking at familiarity with mathematics. On average across OECD countries, the effect of cognitive-activation strategies on opportunity to learn mathematics is mixed in advantaged schools. Some strategies are associated with greater familiarity while

■ Figure 2.23a ■

Teachers' use of cognitive-activation strategies and students' performance in mathematics, by schools' socio-economic profile

Change in mathematics score associated with mathematics teachers' use of cognitive-activation strategies, OECD average



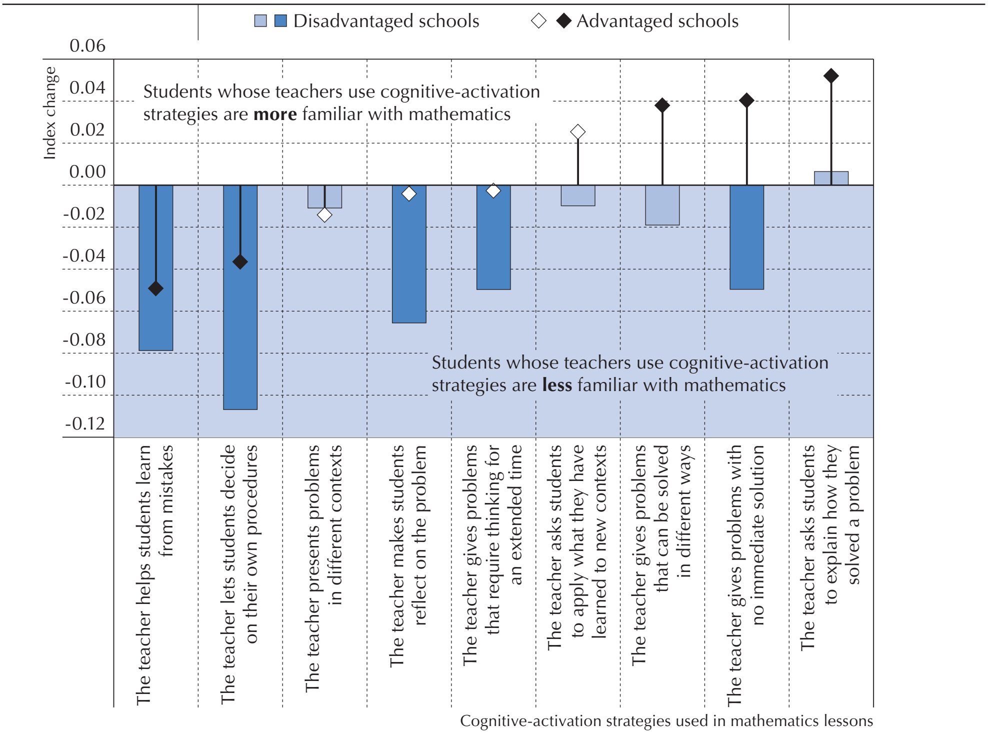
Notes: Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.
Statistically significant values for disadvantaged schools are marked in a darker tone. All values for advantaged schools are statistically significant.
Source: OECD, PISA 2012 Database, Table 2.25b.
StatLink http://dx.doi.org/10.1787/888933377210

other strategies are related to less or no change in familiarity. However, on average across OECD countries, no cognitive-activation strategy is associated with greater familiarity with mathematics among students in disadvantaged schools. Students in disadvantaged schools who are exposed to five out of nine of these strategies are less familiar with mathematics than students who are not exposed; and the remaining four strategies are not related to any significant change in familiarity with mathematics.

■ Figure 2.23b ■

Teachers' use of cognitive-activation strategies and students' familiarity with mathematics, by schools' socio-economic profile

Change in the index of familiarity with mathematics associated with mathematics teachers' use of cognitive activation strategies, OECD average



Notes: Disadvantaged (advantaged) schools are defined as those schools whose average level on the *PISA index of economic, social and cultural status* (ESCS) is statistically significantly below (above) the average across all schools in the country/economy.
Statistically significant values are marked in a darker tone.
Source: OECD, PISA 2012 Database, Table 2.25c.
StatLink *http://dx.doi.org/10.1787/888933377223*

Overall, these results suggest that teachers use cognitive-activation strategies to deepen the curriculum content and support the development of problem-solving abilities among students in advantaged schools. By contrast, in disadvantaged schools, it appears that there might be a price to pay for using strategies that emphasise thinking and reasoning for an extended time: less material is covered.

Why is it so difficult to use cognitive-activation strategies in disadvantaged schools? One reason is that these strategies might be more effective with students who already have a sound background in conceptual and procedural mathematics. Another reason might be that teachers may not be able to make students reflect on problems or assign problems that require thinking for an extended time in classrooms where there is noise and disorder. These difficulties should not discourage mathematics teachers in disadvantaged schools from adopting cognitive-activation strategies and problem solving. The time cost of these strategies can in fact be minimised by choosing well-framed problems and encouraging positive classroom behaviour.

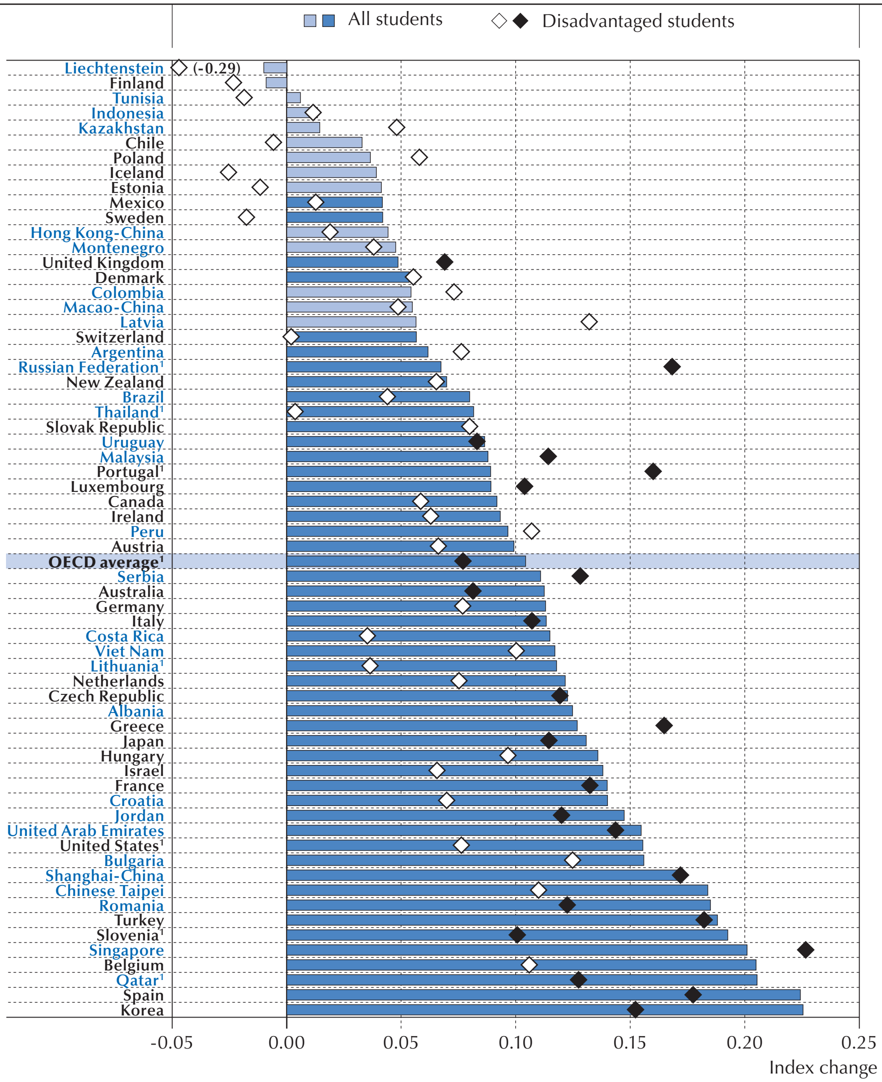
Previous PISA analysis showed that the disciplinary climate is positively correlated to a school's socio-economic profile (OECD, 2013b). In most countries and economies, better disciplinary climate is related to greater familiarity with mathematics, even after comparing students and schools with similar socio-economic profiles (Table 2.26). Moreover, Figure 2.24 shows that, on average across OECD countries, the association between disciplinary climate and familiarity with mathematics is weaker among disadvantaged students than among students in general, possibly because students with more positive attitudes towards mathematics benefit more from a favourable learning environment.

The preceding analyses show that access to mathematics is unequally distributed across individuals, schools and systems. Familiarity with mathematics is strongly related to students' socio-economic status, and the organisation of most education systems tends to reinforce socio-economic inequalities in access to mathematics. Selecting students into more homogenous groups through grade repetition, between-school tracking, academic admission requirements and school transfers is associated not only with a more unequal achievement distribution, but also with more unequal access to mathematics content, on which mathematical literacy is based. This suggests that alternative and more individualised approaches should be considered to provide struggling students with instruction tailored to their ability and needs (see Chapter 5).

■ Figure 2.24 ■

Disciplinary climate and familiarity with mathematics, by students' socio-economic status

Change in the index of familiarity with mathematics associated with a one-unit change in the index of disciplinary climate



1. The difference between disadvantaged and all students is statistically significant.

Notes: The *index of disciplinary climate* is based on students' reports of the frequency with which interruptions occur in mathematics class. Higher values on the index indicate a better disciplinary climate.

Statistically significant values are marked in a darker tone.

Countries and economies are ranked in ascending order of the change in familiarity with mathematics associated with a one-unit change in the index of disciplinary climate *for all students.*

Source: OECD, PISA 2012 Database, Table 2.26.

StatLink http://dx.doi.org/10.1787/888933377232

Notes

1. As discussed in Chapter 1, PISA 2012 data offer various measures of exposure to mathematics and familiarity with mathematics. Most of the analysis presented in Chapter 2 focuses on familiarity with mathematics because it better reflects the effect of cumulative opportunities to learn over students' school career – rather than just recent exposure – and because PISA gauges familiarity over a larger set of items than exposure, offering more statistical robustness and variation.

2. The relatively large within-school variations in the *index of familiarity with mathematics* might be partly explained by a certain degree of subjectivity in the interpretation of the questions, for example in how students define what is frequent and what is rare.

3. Equity in access to mathematics (the percentage of the variation in familiarity explained by students' and schools' socio-economic profile) is computed through a single-level linear regression for consistency with the definition of equity in education (the percentage of the variation in mathematics performance explained by students' socio-economic status) used in previous analyses of PISA 2012 data (OECD, 2013a). The correlation between equity in access to mathematics content and system-level indicators of stratification presented in this chapter is robust to an alternative definition of equity computed through a two-level model (as in Figure 2.1).

4. The OECD Teaching and Learning International Survey (TALIS) is conducted among teachers and leaders of mainstream schools in representative samples of schools. In 2013, 34 countries surveyed teachers in their primary, lower secondary and upper secondary schools. TALIS asks teachers and schools about their working conditions and learning environments. It covers such themes as initial teacher education and professional development; what sort of appraisal and feedback teachers get; the school climate; school leadership; and teachers' instructional beliefs and pedagogical practices. In 2013, some countries also chose to gain additional insights by conducting the survey in schools that participated in the 2012 Programme for International Student Assessment (PISA).

5. The difference in the impact of socio-economic status as students progress through school is probably underestimated, because we cannot observe those students who drop out of school between lower and upper secondary school. Dropout rates are generally much higher among relatively disadvantaged students.

6. These results need to be interpreted with some caution because it is not possible to distinguish whether the association is due to a genuine effect of teaching on achievement, or whether it is due to different uses of these teaching strategies according to students' abilities.

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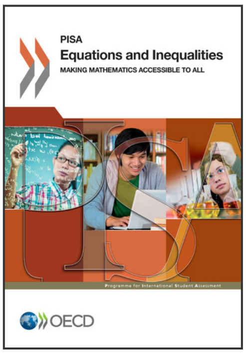

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