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Measuring the impact of structural reforms and investment policies: A DSGE model for South Africa

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MEASURING THE IMPACT OF STRUCTURAL REFORMS AND INVESTMENT POLICIES: A DSGE MODEL FOR SOUTH AFRICA

ECONOMICS DEPARTMENT WORKING PAPERS No. 1747

By Falilou Fall and Paul Cahu

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Abstract/Resumé

Measuring the impact of structural reforms and investment policies: A DSGE model for South Africa

This paper aims at quantifying the macroeconomic and distributional impacts of product market reforms and additional public investment using a DSGE model. The model reflects specific features of the South African economy. Tradable and non-tradable product markets are modelled separately, and a segmented labour market is designed to reproduce the labour market duality in South Africa between skilled and unskilled workers. The role of public investment on total factor productivity and its financing modality are taken into allowing the quantification of the net benefits of reforms.

Our results show that enhancing competition in the non-tradable sector has a short run recessionary impact while deregulating the tradable sector is expansionary. Overall, the latter has a bigger impact on GDP. From a distributional perspective, a product market reform in both sectors benefits all income deciles. Finally, additional public infrastructure investment, either financed by raising VAT or capital income tax, increases GDP in the short-term less than product market reform in the tradable sector but is more expansionary in the long run, so a combination of both reforms would boost living standards.

JEL codes: D24, E24, F13, F14, F41, J64, L11, L51

Key words: Labour market, Productivity, Product market, Structural reforms, Public investment

This Working Paper relates to the 2022 OECD Economic Survey of South Africa.

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Mesurer l'impact des réformes structurelles et des politiques d'investissement : Un modèle DSGE pour l'Afrique du Sud

Ce document vise à quantifier les impacts macroéconomiques et distributifs des réformes du marché des produits et des investissements publics supplémentaires en utilisant un modèle DSGE. Le modèle reflète les caractéristiques spécifiques de l'économie sud-africaine. Les marchés de produits négociables et non négociables sont modélisés séparément, et un marché du travail segmenté est conçu pour reproduire la dualité du marché du travail en Afrique du Sud entre les travailleurs qualifiés et non qualifiés. Le rôle de l'investissement public sur la productivité totale des facteurs et son mode de financement sont pris en compte pour permettre la quantification des bénéfices nets des réformes.

Nos résultats montrent que le renforcement de la concurrence dans le secteur non échangeable a un impact récessif à court terme tandis que la déréglementation du secteur échangeable est expansive. Globalement, cette dernière a un impact plus important sur le PIB. D'un point de vue distributif, une réforme du marché des produits dans les deux secteurs profite à tous les déciles de revenus. Enfin, des investissements publics supplémentaires dans les infrastructures, financés par une augmentation de la TVA ou de l'impôt sur le revenu du capital, augmentent moins le PIB à court terme que la réforme du marché des produits dans le secteur des biens échangeables, mais sont plus expansionnistes à long terme, de sorte qu'une combinaison des deux réformes améliorerait le niveau de vie.

Codes JEL : D24, E24, F13, F14, F41, J64, L11, L51

Mots clés : Marché du travail, Productivité, Marché des produits, Réformes structurelles, Investissement public

Ce Document de travail a trait à l'Étude économique de l'OCDE d'Afrique du Sud.

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Measuring the impact of structural reforms and investment policies: A DSGE model for South Africa

Paul Cahu and Falilou Fall[1]

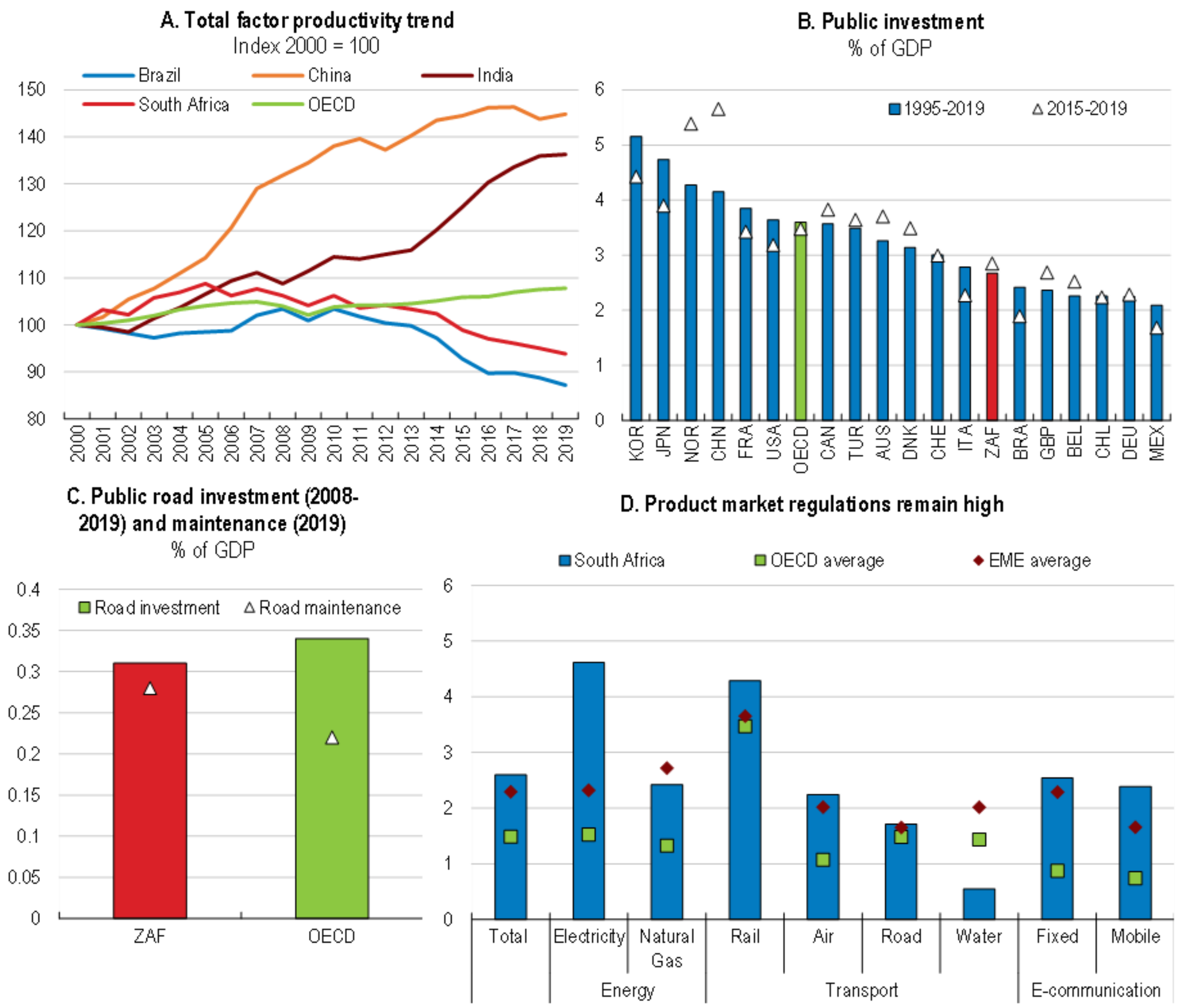
Introduction

Growth-enhancing structural reforms and policies to boost productivity are key to lift living standards. Low growth in the aftermath of the global financial crisis coupled with an ageing population in advanced countries have drawn the focus on ways to increase potential growth, in particular labour market policies and pro-competition measures.

South Africa is no exception. The South African economy has been underperforming in the last decade. Over the period 2009-2019, average annual GDP growth was only 1.7%, while it reached 2.2% for the OECD and 5.4% for other BRIICS countries (Brazil, Russia, India, Indonesia, China and South Africa). Productivity declined during the last decade due to low investment, deteriorating quality of infrastructure and high barriers to entry in product markets and network industries leading to low competition (Figure 1).

[1] Paul Cahu is an external consultant and Falilou Fall is member of the OECD Economics Department. The authors would like to thank Pierre Beynet, Alvaro Pereira, Isabell Koske, Enes Sunel, Sebastien Turban (OECD Economics Department) and Elisa Lanzi and Ioannis Tikoudis (OECD Environment Directorate), for their valuable comments and feedback. Special thanks to Tony Huang for statistical assistance and Emily Derry for editorial assistance (OECD Economics Department).

Figure 1. The South African economy has been underperforming



Note: "EME average" in Panel D refers to the average of Chile, Colombia, Mexico, Turkey, Argentina, Brazil, Indonesia and Russia. Panel D: PMR Index scale from 0 to 6, from most to least competition-friendly regulations.
Source: Penn World Tables; OECD Productivity database; OECD PMR database; OECD calculations.

The goal of this paper is to provide a quantitative assessment of the macroeconomic and distributional impacts of structural reforms and public investment in infrastructure using a dynamic general equilibrium economy framework. Potential losers of structural reforms often oppose any reform as they are affected by short run costs associated with it. However, an important part of the benefits of structural reforms runs through general equilibrium effects on prices and their second-round effects on households and markets equilibrium. Therefore, it is important to determine both short-run costs associated with product market reforms and their distributional impacts.

Our contribution to the literature is three-fold. First, we add to the literature on the short-run effects of product market reforms by distinguishing reforms affecting sectors exposed to international competition (tradable) and domestic sectors (non-tradable) in a model that captures empirical features of product and labour markets. We are therefore able to show that product market reforms have different short-run effects whether they are applied to the tradable or non-tradable sector. Second, by including the financing instrument of public investment, this paper sheds some light on the net impact of increasing public investment spending. The model shows that product market reforms affecting the tradable sector have larger short-run effects than increasing public spending, nonetheless, in the long run, this is reversed. Third, it is also the first model in a DSGE framework, in our knowledge, with a disaggregation of households into income deciles allowing capturing some distributional effects of reforms.

To this end, we develop an open-economy general equilibrium framework embedding features of South African product and labour markets. Product markets are characterised by monopolistically competitive firms with positive mark-ups (Bilbiie et al., 2012 and Andrés et al., 2017). Three sub-labour markets (high-skilled, formal low-skilled and informal low-skilled) and a Nash-bargaining wage setting in each sub-market capture the segmented South African labour market. The model considers search and matching frictions, which are different between sub-markets. Two types of consumers, the liquidity-constrained and the liquidity-unconstrained, are considered, as in Andrés et al. (2017), allowing to replicate savings/investment dynamics and income distribution. The liquidity-constrained households with a “Keynesian” profile consume all their income including transfers from the government. The liquidity-unconstrained, the so-called *Ricardian* consumers optimise the allocation of their revenues between consumption and savings considering their assets portfolio. Finally, the role of government spending on total factor productivity through public investment and its financing structure is explicitly modelled allowing capturing the net benefits of reforms.

In particular, given very high-income inequalities and capital concentration in South Africa, the model assumes as in the DSGE literature that the share of liquidity-constrained agents, called “constrained” is exogenous and fixed. The current account and the fiscal account are balanced in the long run. The current account is stabilised by an adjustment of the real exchange rate, ensuring that the external debt as a share of GDP remains exogenous in the long run. This condition implies in return that nominal exports are evolving in line with nominal imports. The economy functions then as if the optimising agents would adjust their consumption to ensure the trade balance, as is the case in most open-economy DSGE. The fiscal account is balanced thanks to a fiscal rule ensuring that the debt to GDP ratio remains constant in the long run. We calibrate the model to match South African macroeconomic data and show that the model successfully reproduces several features of the business cycles and the income and wealth distributions.

The product market reform consists in a reduction in barriers to entry by cutting mark-ups by 10% separately in tradable and non-tradable sectors. The public investment policy consists in increased public infrastructure spending by 1% of GDP financed either by increasing VAT tax rate or by increasing the capital income tax rate.

The findings of the paper are manifold. First, we reconcile two findings in the literature regarding the short-run effects of product market reforms. We find that deregulating product markets is recessionary in the short run in the non-tradable sector as shown for instance in Cacciatore and Fiori (2016). But the product market reform of the tradable sector is expansionary even in the short run, as found by Andrés et al. (2017). The main difference is that these papers do not distinguish between tradable and non-tradable sectors in their reform. These two sectors face different types of competition and market structures that could justify differences in the impact of reform. When the reform targets non-tradable sectors (such as professional and business services or telecommunications), in the face of falling mark-ups, investment decreases and job destruction is high in the short run as the reallocation of resources from incumbents to new entrants is slow. In contrast, opening more the tradable sector to new entries attracts immediately an inflow of investment, hinging on the expected competitiveness and export gains, which boosts activity in the short run. In the long run, product market opening in both sectors has a positive impact on GDP but the effect of reforming the tradable sector is higher than reforming the non-tradable sector.

Second, the distributional impacts of product market reforms are striking. In the medium to long run, the gains in income of reforming product markets in the non-tradable sector are quite evenly distributed among deciles 1 to 9 while they are unevenly distributed among deciles when reforming the tradable sector. In particular, the top 0.01% are net losers from the tradable sector reform both in the short and the long run.

By contrast, middle-income earners (from decile 5 to 9) benefit the most from the reform of the tradable sector, while low-income earners, though benefiting from the reform, gain less than middle income earners. The top income earners (0.01%) lose from the reform in the long run, but their losses are more limited than in the reform of the non-tradable sector.

Finally, increasing public infrastructure investment financed either by raising the value-added taxes or the capital income taxes (CIT) has a lower short run effect on GDP than reforming the tradable sector but has a higher expansionary impact in the long run. Because of its size, improving competition in the tradable sector drives a larger response in investment than a public sector infrastructure investment (1 percentage point of GDP) in the short run. The output impact of increasing public investment is similar whether financed by VAT or capital income taxes. In the short run, the aggregate demand effect of increasing public investment dominates the negative price effect on consumption of increasing VAT or capital income tax. However, consumption increases more when the reform is VAT financed than when it is capital income tax financed. A capital income tax affects only Ricardian households and pushes them to boost their investment with expected gains linked to improving total factor productivity while the increase in disposable income in the short run is lower than in a VAT financed reform. This result is likely linked, on the one hand, to the high inequality in income distribution in South Africa, whereby the share of the income of the bottom half of income earners is low. On the other hand, the structure of VAT exemptions and reduced rates diminish the elasticity of consumption to VAT increase for low-income households. Moreover, increasing capital income tax to finance public investment reduces more income inequalities than increasing VAT.

Our paper contributes to a large and varied literature on the macroeconomic consequences of product and labour market regulation reforms (den Haan et al., 2000; Veracierto, 2008; Bilbiie et al. 2012; Ghironi and Melitz, 2005; and Jaimovich and Floetotto, 2008; Petrosky-Nadeau and Wasmer, 2011; Eggertsson et al., 2014). Another strand of the literature differentiates between the short- and long-run consequences of market reforms (Blanchard and Giavazzi, 2003). The closest study to the current paper in this literature is Cacciatore and Fiori (2016) that analysed, in a DSGE framework, the short run and transition dynamics of the macroeconomic impacts of product and labour market reforms. They find that reforms have short-run recessionary impacts, nonetheless, increase employment and output in the long run. In the empirical literature, Causa et al. (2016) present the closest analysis in terms of identifying the impact of structural reforms on different segments of the income distribution. They find that income gains associated to easing barriers to firm entry and competition in product markets accrue to households at large and are equally shared.

Our model differs from the literature by explicitly including the role of government spending and taxation and different types of households and a wealth accumulation process. The seminal contribution by Galí et al. (2007) embedded rule-of-thumb (or non-Ricardian) agents in a standard monetary New Keynesian model to better account for the role of government spending. Mourougane and Vogel (2008) also considers liquidity-constrained and unconstrained households and analyse the differentiated impacts of tax and labour market reforms on these two categories of households. Andrés et al. (2017) also incorporate constrained and unconstrained households with different financial investment roles to unveil the financial mechanism of reforms. We complement this literature by making it key to match wealth accumulation and income distribution. Moreover, to our knowledge, this is the first paper tracking in a DSGE model the distributional impacts of product market reforms.

Another strand of the literature incorporates explicitly government investment in DSGE model to analyse its macroeconomic impact and the crowding out effect on private investment and household consumption (Dupaigne and Fève, 2016; Straub and Tchakarov, 2007 among others). Unlike these models, our modelling focuses on the role of public infrastructure in total factor productivity and not on the persistence of public investment and its multiplier effect. The closest paper in this literature is Hickey et al. (2019), which analyses the macroeconomic impact of increasing government investment and public capital stock considering the different financing strategy. Hickey et al. (2019) find that the least harmful way of financing government investment, which preserves both fiscal and external balances, is by reducing other government spending. Financing government investment with debt instead, worsens fiscal and external balances. Financing investment with labour taxes reduces the external balance, while financing with VAT only does so in the very short run. Our modelling compares these different channels but also considers an increase in capital income tax.

In our model, we do not consider a joint reform of both product and labour markets, though we model explicitly how the dual South African labour market is functioning. The labour market is characterised by huge differences in earnings between skilled and unskilled workers and is modelled by three connected sub-markets allowing a replication of the labour income distribution. Our modelling of the segmentation of the labour markets of skilled and unskilled workers can be related somehow to papers explicitly including an informal sector. Munkacsi and Sexegaard (2017) for South Africa and Charlot et al. (2015) for Brazil propose a DGSE model with a distinction between formal and informal sectors to analyse the impact of product and labour market reforms. The functioning of our non-tradable sector and its linkages with the tradable sector and the hiring of unskilled workers can easily be extended to an informal sector analysis.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 describes the calibration and discusses the performance of the model in relation to the data. Section 4 studies the implications of product market reforms. Section 5 discusses the impact of public investment depending on its financing. Section 6 concludes.

The model

The model structure is complex, embedding multiple productive sectors, several linkages between labour markets and endogenous total factor productivity. Therefore, the model dynamics have been approximated around a steady state. In this approach, all equations are log-linearised around the steady state. First-order equations use steady-state volumes and relative prices. These quantities have been calibrated from the 2015 input-output tables. For the sake of explanation, some equations are presented and derived in levels but then further log-linearised for estimation and simulation purposes. In all what follows, the reference price is the production price in the tradable sector and all other prices are expressed as a share of this price.

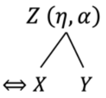
Production

There are three economic sectors in the model: one private sector for a tradable product (sector 1), one private sector for a non-tradable product (sector 2) and one public sector (sector 3). This structure is needed to reflect both the dependence of the South African economy to its diversified export sector as well as the large size of the public sector and its impact on productivity through public investments in infrastructure (transport and energy especially) and public services (such as health and education). Level variables are denoted in capital letters and log-linearised variables in lower case.

Substitution between products is described by standard CES-utility functions calibrated for South Africa. Aggregate investment, intermediate inputs and consumption are each CES aggregations of tradable, non-tradable, domestic, and imported goods as imperfect substitutes. For ease of presentation, we introduce the following scheme to represent such aggregation, where Z is the composite, X and Y are the element, σ is the elasticity of substitution and α is the share of the element X in value in the composite in the reference year (in which all prices equal one), see Figure 2.

Figure 2: CES aggregation in the model

$$Z = \left(\alpha^{\frac{1}{\sigma}} X^{1-\frac{1}{\sigma}} + \left(1-\alpha^{\frac{1}{\sigma}}\right) Y^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \Leftrightarrow$$



Value-added and prices

Firms in sector *j* produce in each period t a quantity $Y_{j,t}$ of goods and services using an aggregate capital $K_{j,t-1}$, an aggregate labour of low-skilled and high-skilled labour $L_{j,t}$ and intermediate inputs $Z_{j,t}$. Producers do not use inputs from the public sector. Labour productivity of the sector is denoted by $A_{j,t}$. A Leontief production function is used to overcome the lack of macroeconomic data to estimate potential price effects on the substitution between capital and labour on the one hand and intermediate inputs substitutions on the other hand.

$$Y_{j,t} = \min\left\{\frac{A_{j,t}K_{j,t-1}^{\gamma_j}L_{j,t}^{1-\gamma_j}}{1-\psi_j}, \frac{Z_{j,t}}{\psi_j}\right\} \qquad (1)$$

There is a continuum of producers in each sector, which operate in an imperfect competition framework. All producers have the same marginal cost MC_t^j. The optimisation of the intermediate input $Z_{j,t}$ gives, with Leontief functions:

$$Z_{j,t} = \psi_j Y_{j,t} \qquad (2)$$

$$Y_{j,t} = \frac{A_{j,t}K_{j,t-1}^{\gamma_j}L_{j,t}^{1-\gamma_j}}{1-\psi_j} \qquad (3)$$

Producers purchase the intermediate input, which is a composite of imported goods, domestic tradable goods (of sector 1) and non-tradable services (of sector 2) and transform it into final good varieties. Assuming Calvo (1983) price-setting, an exogenous share α_P of producers can reset their prices in each period to the value $P_t^{\#}$ that maximise their profits. When they cannot, the price is modified as an average of the past price (backward-looking indexation) and what the steady-state (the central bank target) inflation rate, $\bar{\pi}$ would require.

$$P_t = \begin{cases} P_{t-1}(1 + \rho_P\pi_{t-1} + (1-\rho_P)\bar{\pi}) & if \quad price\ not\ reset \\ P_t^{\#} & if \quad price\ reset \end{cases} \qquad (4)$$

Accordingly, price dynamics in sector 1 (tradable) is given by the following dynamics (log-linearised), where mc is the log-linearised marginal cost, π is the spread to the stationary inflation rate, θ_1 is the elasticity of substitution of the varieties and θ_1^* this same elasticity at the steady state. The mark-up rate is given by $\frac{\theta_1}{\theta_1-1}$. The discount factor denoted β, the share of the producers being able to reset their prices at each period denoted α_P and the indexation parameter of past prices denoted ρ_P allow to compute the Phillips curve:

$$\pi_{1t} = \rho_P\pi_{t-1} + \beta(\pi_{1t+1} - \rho_P\pi_{1t}) + (1-\beta\alpha_P)\frac{1-\alpha_P}{\alpha_P}\left(mc_t^1 - \frac{\theta_1-\theta_1^*}{\theta_1^*(\theta_1^*-1)}\right) \qquad (5)$$

We define price dynamics as well in sector 2 (non-tradable) as: $\pi_{2,t} = \pi_t + q_{2,t} - q_{2,t-1}$, where $q_{2,t}$ is the relative price in sector 2 (log-linearised). The dynamics of inflation in sector 2 is then:

$$\pi_{2,t} = \rho_{2,P}\pi_{2,t-1} + \beta(\pi_{2,t+1} - \rho_{2,P}\pi_{2,t}) + (1-\beta\alpha_{2,P})\frac{1-\alpha_{2,P}}{\alpha_{2,P}}\left(mc_t^2 - \frac{\theta_2-\theta_2^*}{\theta_2^*(\theta_2^*-1)} - q_{2,t}\right) \ (6)$$

In the public sector, there is no profit and therefore the marginal cost equals the price:

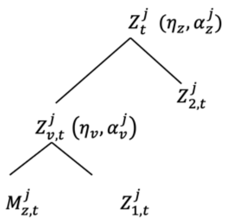
$$q_{3,t} = mc_t^3 \qquad (7)$$

Intermediate inputs

We consider an aggregate of intermediate inputs in quantity Z_t^j built as a twice-nested CES function of a mix of imported goods used as inputs, $M_{z,t}^j$ and domestic tradable good in quantity $Z_{1,t}^j$ on the one hand

and domestic non-tradable goods and services in quantity $Z_{2,t}^j$ on the other hand. This aggregate intermediate input is defined by the following tree, see Figure 3:

Figure 3: Structure of aggregate input



Using the properties of the CES aggregation, one can compute the demand of the three types of inputs from the desired level of production and the price of these inputs assuming that costs are minimized by the producers. These demand functions can be written as log-linear approximations around the initial steady state (see technical appendix for derivations), where the time subscripts have been omitted for simplicity and where q_z^j is the price of the aggregate, q_v^j is the price of the aggregate of imported and domestic goods (tradable), $q^\star$ is the price of imports and q_2 the price of non-tradable products[2], $\Delta\tau_1^z$, $\Delta\tau_1^z, \Delta\tau_\star^z$ are the variations in the effective tax rate[3] of respectively the domestic tradable input, imported inputs and non-tradable inputs.

$$\begin{cases} z_2^j = y^j + \eta_z\left(q_z^j - q_2 - \Delta\tau_2^z\right) & (8a) \\ z_1^j = y^j - \eta_v \Delta\tau_1^z + \eta_z q_z^j & (8b) \\ m_z^j = y^j - \eta_v \Delta\tau_1^z + \eta_z q_z^j + (\eta_v - \eta_z) q_v^j - \eta_v (q^\star + \Delta\tau_\star^z) & (8c) \end{cases}$$

The prices of aggregates can be deduced as follows:

$$\begin{cases} q_v^j = \alpha_v^j \Delta\tau_1^z + \left(1 - \alpha_v^j\right)(q^\star + \Delta\tau_\star^z) & (9a) \\ q_z^j = \alpha_z^j q_v^j + \left(1 - \alpha_z^j\right)(q_2 + \Delta\tau_2^z) & (9b) \end{cases}$$

The labour market and wage setting

To be able to track the impact of product market reforms on the distribution of households' income, the model considers two types of workers in three distinct labour markets. Workers who completed secondary education are considered as high-skilled, while the rest of the active population is assumed to be low-skilled. High-skilled workers are assumed to be hired from a closed labour market denoted by A and by the three sectors. Low-skilled workers can work in either the submarket B, which pools hirings from the tradable sector and the public sector or the submarket C, which is related to the non-tradable sector, see Figure 4. Labour productivity of low-skilled workers is much lower in the private non-tradable sector in South Africa, which is largely informal (see the calibration section). This distinction is therefore necessary to capture asymmetries in the evolution of real wages in the tradable and non-tradable sector. We consider search and matching frictions on the labour market, to capture the large unemployment rate in South Africa, especially in the informal low-skilled market. The labour market is modelled using a wage-bargaining model. Wages are determined as a Nash equilibrium between the employers who minimise their hiring costs and workers who maximise the surplus they gain from employment. In the long run, wage levels are basically a weighted average of the worker's reservation wage, which differs between workers of the three different submarkets and the hiring cost, which depends on the tensions in the labour market. The weights

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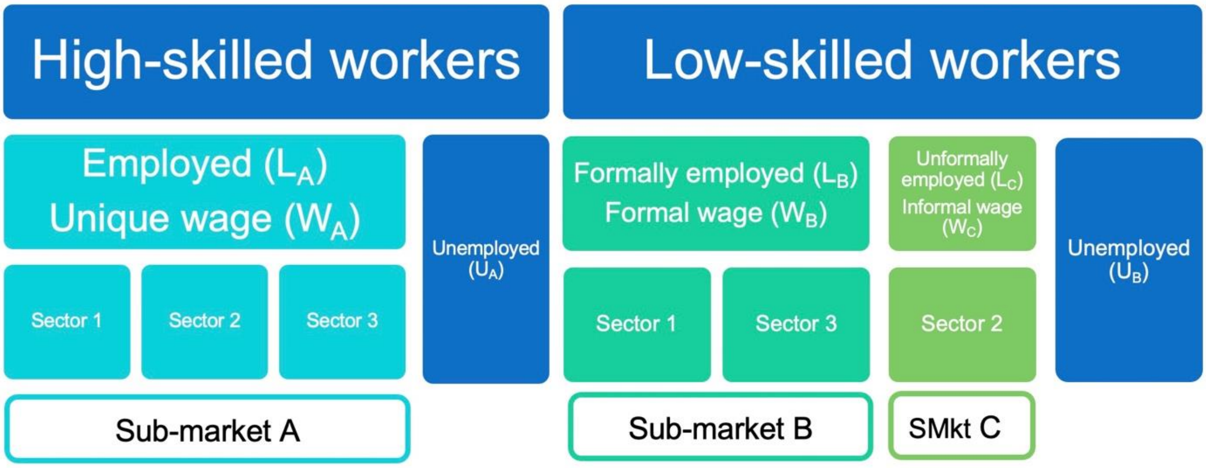
[2] All prices are divided by the price of domestic tradable goods which is the reference price.

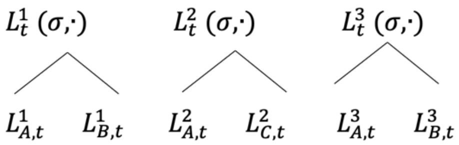
[3] $\Delta\tau \equiv \frac{\tau - \tau^*}{1+\tau^*}$, where τ^* is the tax rate at the initial steady state.

are related to the exogenous bargaining power which relates to characteristics of the work contract influenced by unionisation and informality among other factors.

Figure 4: Labour sub-markets



Producers hire low-skilled and high-skilled workers. The labour aggregate in each sector is a CES combination of the two types of labour, with the same constant elasticity, denoted σ, for the three sectors. $L^j_{A,t}$ denotes the demand of high-skills labour in sector *j* (in level), $L^j_{B,t}$ the demand for low-skills labour in the formal market (for the public sector and the tradable sector firms) and $L^j_{C,t}$ the demand for low-skills labour in the informal market (for the non-tradable firms):



Hiring costs

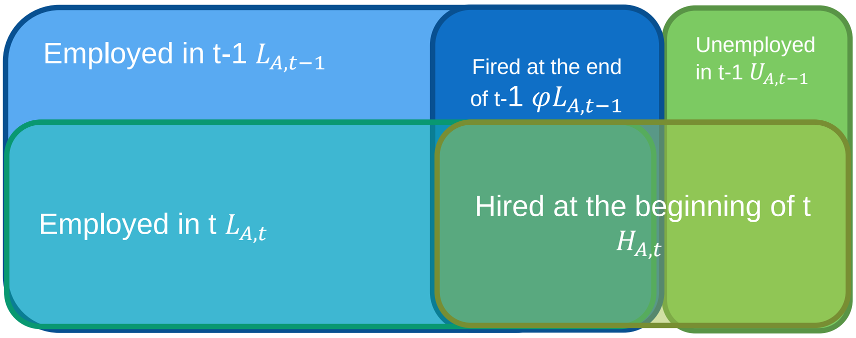
Producers face real hiring costs $HC_{i,t}$, linked to hirings for both types of labour $H^j_{i,t}$, with $i = \{A, B, C\}$ designating the labour submarket. The profit Π^j_t of producers in sector *j* is given by:

$$\Pi^j_t = \left(MC^j_t - \psi_j Q^j_{Z,t}\right)Y_{j,t} - \sum_{i\in\{A,B,C\}} W_{i,t}L^j_{i,t} - \sum_{i\in\{A,B,C\}} HC_{i,t}H^j_{i,t} - R_tK_{j,t} \quad (10)$$

The dynamics of the employed labour is, as follows, for $i = \{A, B, C\}$, with an exogenous rate of attrition (or firings) $\varphi_i$of current employees:

$$L_{i,t} = (1 - \varphi_i)L_{i,t-1} + H_{i,t} \quad (11)$$

Figure 5: Van Allen figure of labour status change between two periods



The probabilities to be hired P_t^i are therefore defined as follows with H_t^A the number of new hirings in each period, U_{t-1}^A the number of unemployed in the previous period and L_{t-1}^A the number of employed in the previous period. The equation describes that newly hired people in the current period are randomly drawn from people who lost their job at the previous period and those who were unemployed, see Figure 5.

$$p_t^A = \frac{H_{A,t}}{U_{A,t-1} + \varphi_t^A L_{A,t-1}} \qquad (12a)$$

$$p_t^B = \frac{H_{B,t}}{U_{B,t-1} + \varphi_t^B L_{B,t-1} + \varphi_t^C L_{C,t-1}} \qquad (12b)$$

$$p_t^C = \frac{H_{C,t}}{U_{B,t-1} + \varphi_t^B L_{B,t-1} + \varphi_t^C L_{C,t-1}} \qquad (12c)$$

The hiring costs are defined as follows, as in Blanchard and Giavazzi (2003):

$$HC_{i,t} = \beta_t^i \sqrt{p_t^i} \qquad (13)$$

Factor demand

The producers profit maximisation is given by:

$$\max_{K_{t-1}, L_{A,t}, L_{B,t}, L_{C,t}, H_{A,t}, H_{B,t}, H_{C,t}} \mathbb{E}\left[\sum_{k=0}^{\infty} \rho_{t,t+k} \Pi_{t+k}\right] \qquad (14)$$

under the constraints (3), (4), (5) and (6) with $\rho_{t,t+k}$ denoting the k-periods ahead stochastic discount factor of Ricardian households. The capital demand $K_{j,t}$ is related to the real cost of capital R_t and the marginal production cost MC_t^j:

$$R_t = \gamma_j \left(MC_t^j - \psi_j Q_{Z,t}^j\right) \frac{Y_{j,t}}{K_{j,t}} \qquad (15)$$

The aggregated labour demand L_t^j is given for both low- and high-skilled labour $i = \{A, \{B \text{ or } C\}\}$ as a function of the real wage in these submarkets $W_{i,t}$ and hiring costs.

$$(1-\gamma_j)\left(MC_t^j - \psi_j Q_{Z,t}^j\right) \frac{Y_{j,t}}{L_{j,t}} \left(\alpha_L^j \frac{L_t^j}{L_{i,t}^j}\right)^{\frac{1}{\sigma}} = W_{i,t} + HC_{i,t} - \mathbb{E}\left[\rho_{t,t+1}\left(1-\varphi_t^i\right) HC_{i,t+1}\right] \qquad (16)$$

Wage bargaining

On labour market A, let us denote $V_{A,t}$ the value of being employed and $V_{U,t}$ the value of being unemployed, as a function of reservation wage $W_{A,t}^U$. The value of being employed is the wage that comes from employment plus the value that derives from the opportunity to both retain a job in the next period or to lose a job but get rehired. The value of being unemployed is the sum of the reservation wage plus the value derived from the opportunity to get a job in the next period or remaining unemployed. This form follows from the assumption of rational expectations of workers where the mathematical expectancy of the sum of expected values from the current time to eternity can be rewritten as a linear function of the contemporaneous value and the expected value in the next period. These values can be computed as:

$$\begin{cases} V_{A,t} = W_{A,t} + \mathbb{E}\left[\rho_{t,t+1}\left(1-\varphi_{A,t+1}\right)(1-p_t^A)V_{A,t+1} + \varphi_{A,t+1}(1-p_{t+1}^A)V_{U,t+1}\right] & (17a) \\ V_{U,t} = W_{A,t}^U + \mathbb{E}\left[\rho_{t,t+1}\left((1-p_t^A)V_{U,t+1} + p_{t+1}^A V_{A,t+1}\right)\right] & (17b) \end{cases}$$

The Generalised Nash bargaining is, with the surplus for the producer $J_t = HC_{i,t}$:

$$\max_{W_{A,t}} \left(V_{A,t} - V_{U,t}\right)^{\chi^A} J_t^{1-\chi^A} \qquad (18)$$

This gives the wage-setting equation:

$$V_{A,t} - V_{U,t} = \frac{\chi^A}{1-\chi^A}(1-\tau_A^L)J_t \qquad (19)$$

Plugging the definition of the worker's surplus into this expression gives the wage target $\widetilde{W}_{A,t}$ dynamics in the labour market A. The net wage equals the sum of the reservation wage plus a share[4] of the hiring cost, minus the value linked to being employed in the future.

$$\begin{aligned} \left(1-\tau_{A,t}^L\right)\widetilde{W}_{A,t} = {} & W_{A,t}^U + \frac{\chi_t^A}{1-\chi_t^A}\left(1-\tau_{A,t}^L\right)HC_t^A \\ & - \mathbb{E}\left[\rho_{t,t+1}\left(\frac{\chi_{t+1}^A}{1-\chi_{t+1}^A}\right)\left(1-\tau_{A,t+1}^L\right)\left(1-\varphi_{A,t+1}\right)(1-p_{t+1}^A)HC_{t+1}^A\right] \quad (20) \end{aligned}$$

In the two other labour markets, workers can transition between submarkets B and C. The expression of the target wages is, for $\{i,-i\} = \{B,C\}$. The net wage must equal the reservation wage plus a share of the hiring cost, minus the value associated with being employed in the future plus an additional component which is linked to the value of switching from the informal to the formal market.

$$\begin{aligned} \left(1-\tau_{i,t}^L\right)\widetilde{W}_{i,t} = {} & W_{i,t}^U + \frac{\chi_t^i}{1-\chi_t^i}\left(1-\tau_{i,t}^L\right)HC_t^i \\ & - \mathbb{E}\left[\rho_{t,t+1}\left(\frac{\chi_{t+1}^i}{1-\chi_{t+1}^i}\right)\left(1-\tau_{i,t+1}^L\right)\left(1-\varphi_{i,t+1}\right)\left(1-p_{t+1}^i\right)HC_{t+1}^i\right] \\ & + \mathbb{E}\left[\rho_{t,t+1}\left(\frac{\chi_{t+1}^{-i}}{1-\chi_{t+1}^{-i}}\right)\left(1-\tau_{-i,t+1}^L\right)\left(1-\varphi_{-i,t+1}\right)\left(1-p_{t+1}^{-i}\right)HC_{t+1}^{-i}\right] \quad (21) \end{aligned}$$

The dynamics of the wage setting is assuming a Calvo-type indexation, in a similar way to prices. The wage target $\widetilde{W}_{i,t}$ may differ from actual wage $W_{i,t}$ because at each period, only a share α_W of working contracts can be reset to match the target wage. The model also accounts for the possibility of a dynamic indexation: a share ρ_W of the indexation is based on the past growth of wages. Once log-linearized around the initial steady state, the wage dynamics become in each of the three labour markets i:

$$\begin{aligned} & w_{i,t} - w_{i,t-1} + \pi_t - \rho_W \pi_{t-1} \\ & \quad = \beta\left(w_{i,t+1} - w_{i,t} + \pi_{t+1} - \rho_W \pi_t\right) + (1-\beta\alpha_W)\frac{1-\alpha_W}{\alpha_W}\left(\widetilde{w}_{i,t} - w_{i,t}\right) \quad (22) \end{aligned}$$

[4] Which increases with the bargaining power of employees.

Moreover, the population participating to the labour market N_t^j responds to the employment rate.

$$N_t^j = \left(N_{t-1}^j\right)^{\varsigma_j} \left(L_{t-1}^j\right)^{\varsigma'_j} \quad (23)$$

Household consumption

There are two types of consumers, the first are liquidity-constrained and consume all their income, Y_c, composed of the transfer Tr_c and the labour income from the three different types of labour. The aggregate consumption of the constrained agents is therefore matching their total income Y_c. The relative population weight of each group of workers is denoted ν_X^c (variable indices have been omitted to ease reading).

$$Y_c = Tr_c + \sum_{X=\{A,B,C\}} \nu_X^c W_X L_X (1 - \tau_c) \quad (24)$$

The second type of consumers are called Ricardian, because they can optimise their intertemporal utility. The program of these optimising agents is as follows. To ensure the stability of the system, we assume that there is a risk premium for foreign debt, which reduces the expected value of the foreign bonds. The bond purchasers do not consider the endogeneity of this risk premium. The level of consumption is denoted by C_t and CH_t denotes the level of consumption habits. The relative price of consumption is $Q_{c,t}$ and the relative price of investment is $Q_{i,t}$. The nominal exchange rate is E_t. Investment is denoted by Inv_t and the private capital stock is K_t. The volumes of domestic and foreign bonds are B_t and B_t^*, respectively. The risk premium of foreign debt increases exponentially with its level (see Gertler et al., 2007). The nominal interest rates are respectively i_t and i_t^* for domestic and foreign bonds. Tradable production prices are respectively P_t and P_t^* for domestic and foreign goods. The average wage and employment of optimising agents are denoted $W_{o,t}$ and L_t^o. Tr_t^o are transfers to these agents. The real GDP growth rate is denoted Γ_t. The model is assuming that the economy is growing around a steady-growth path. By dividing everything by the growth rate, we clean the GDP series from both demographic and total factor productivity growth which are exogenous is this setting.

$$\max_{C_t, Inv_t, K_t} \sum_{t=0}^{\infty} \mathbb{E}[\beta^t \{(1 - \phi^C) \ln(C_t - CH_t)\}] \quad (25a)$$

$$s.t. \quad Q_{c,t} c_t + Q_{i,t} Inv_t + \frac{\phi_K}{2} Q_{i,t} \left(\frac{Inv_t}{K_{t-1}} - \delta + 1 - \Gamma\right)^2 \frac{K_{t-1}}{\Gamma_t} + E_t \frac{P_t^*}{P_t} B_t^* \exp(\eta_b B_t^*) + B_t$$
$$= (1 - \tau^o) W_{o,t} L_t^o + Tr_t^o + \left(1 - \tau_{K,t}\right) R_t \frac{K_{t-1}}{\Gamma_t} + \frac{1 + i_{t-1}}{\pi_t} \frac{B_{t-1}}{\Gamma_t}$$
$$+ e_t \frac{(1 + i_{t-1}^*)}{\pi_t} \frac{P_{t-1}^*}{P_{t-1}} \frac{B_{t-1}}{\Gamma_t} + \Pi_t + HC_t \quad (25b)$$

$$s.t. \quad K_t = (1 - \delta) K_{t-1} + Inv_t \quad (25c)$$

$$s.t. \quad CH_t = \phi_C C_{t-1} \quad (25d)$$

The first order conditions respective to consumption give, by replacing consumption habits by their definition, the value of the shadow price of consumption $\lambda_t Q_{c,t}$:

$$\lambda_t Q_{c,t} = \frac{1 - \phi_C}{C_t - \phi_C C_{t-1}} \quad (26)$$

The respective conditions to the domestic and foreign bonds give:

$$\lambda_t = \beta\, \mathbb{E}\left[\frac{\lambda_{t+1}}{\Gamma_{t+1}} \frac{1+i_t}{\pi_{t+1}}\right] \qquad (27)$$

$$\lambda_t E_t \frac{P_t^*}{P_t} \exp(\eta_b B_t^*) = \beta\, \mathbb{E}\left[\frac{\lambda_{t+1}}{\Gamma_{t+1}} \frac{1+i_t^*}{\pi_{t+1}} E_{t+1} \frac{P_t^*}{P_t}\right] \qquad (28)$$

Combining the two previous equations gives the uncovered interest parity condition. The risk premium does not show up in this equation because it raises the nominal value of foreign bond directly into the budget constraint of the optimizing agent (see equation 25b).

$$\mathbb{E}\left[\frac{1+i_t}{\pi_{t+1}}\right] = \mathbb{E}\left[\frac{1+i_t^*}{\pi_{t+1}} \frac{E_{t+1}}{E_t}\right] \qquad (29)$$

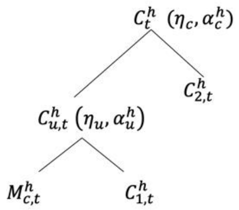
At the steady state, the nominal interest rates are:

$$\frac{1}{1+i} = \frac{1}{1+i^*} = \frac{\beta}{\Gamma\pi} \qquad (30)$$

The level of imported goods for optimising consumers is given by the trade account balance. Imported goods are used by both optimizing (M_c^o) and constrained consumers (M_c^c), for investment in the three sectors $(M_i^j), j = \{1,2,3\}$ and as intermediate input in the three sectors $(M_z^j), j = \{1,2,3\}$.

$$\bar{X}(Q_x)^{1-\eta_x} = M_c^c + M_c^o + M_i^1 + M_i^2 + M_i^3 + M_z^1 + M_z^2 + M_z^3 + B^* \qquad (31)$$

Both types of consumers optimise their consumption basket, which allows setting the optimal prices and the optimal levels. The composite of imported and domestic good 1 is indexed by *u*. We have two types $h = \{o, c\}$ of consumers and we use CES nested utility functions. Exploiting the properties of the CES functions, we can express the demands of the three types of products as a function of their prices and the level of aggregate consumption. We can also compute the aggregate price from the prices of each product.



For liquidity-constrained agents, the aggregate consumption $C_{c,t}^c$ is defined by the nominal income $Y_{c,t}$ and the aggregate consumption price $Q_{c,t}$.

$$C_t^c Q_{c,t}^c = Y_{c,t} \qquad (32)$$

The demand equations can be log-linearized as follows:

$$c_{2,t}^c = y_{c,t} + \eta_c\left(q_{c,t}^c - q_{2,t} - \Delta\tau_{2,t}^c\right) \qquad (33a)$$

$$c_{1,t}^c = y_{c,t} - \eta_u \Delta\tau_{1,t}^C + \eta_c q_{c,t}^c + (\eta_u - \eta_c) q_{u,t}^c \quad (33b)$$

$$m_{c,t}^c = y_{c,t} - \eta_u\left(\Delta\tau_{1,t}^C + q_t^\star\right) + \eta_c q_{c,t}^c + (\eta_u - \eta_c) q_{u,t}^c \qquad (33c)$$

For the optimising agents, we do things in reverse and express the consumption of domestic tradable C_1^o and non-tradable products C_2^o as a function of imports M_c^o and the prices. This gives after log-linearization:

$$c_{1,t}^o = m_{c,t}^o + \eta_u\left(\Delta\tau_{m,t}^c - \Delta\tau_{1,t}^c + q_t^\star\right) \qquad (34a)$$

$$c_{2,t}^0 = m_{c,t}^o + \eta_u \Delta\tau_{m,t}^c - \eta_c \Delta\tau_{2,t}^c + \eta_c q_{2,t} + (\eta_c - \eta_u) q_{u,t}^o + \eta_u q_t^\star \quad (34b)$$

The investors, the Ricardian agents, can save in three distinct types of assets: domestic and foreign bonds and physical capital. The first order condition related to investment, gives the shadow price of the capital stock μ_t:

$$\mu_t = \lambda_t Q_{i,t}\left(1+\phi_K\left(\frac{Inv_t}{K_{t-1}}-\delta\ +1-\Gamma\right)\right) \qquad (35)$$

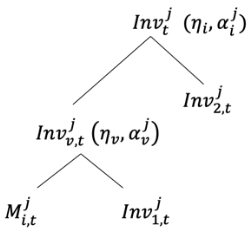
The first order condition related to the capital stock gives the law of motion of the capital price:

$$\mu_t = \mathbb{E}\left[\frac{\beta(1-\delta)}{\Gamma_t}\mu_{t+1}+\frac{\beta\lambda_t}{\Gamma_t}\left((1-\tau_K)R_t-\frac{\phi_K}{2}q_{i,t}\left(\frac{Inv_t}{K_{t-1}}-\delta\ +1-\Gamma\right)^2\right)\right] \qquad (36)$$

At the steady state, this equation gives the value of the real cost of capital:

$$R = q_i\left(\frac{\Gamma}{\beta}-(1-\delta)\right) \qquad (37)$$

The investment good is a composite of imported good, tradable goods of the sector 1 and the non-tradable services of sector 2:



The investment price is therefore linked to import price and price of services. The price of the composite of tradable goods (equipment) $Q_{v,t}^j$ is given as follows, with τ_1^i being the effective tax rate on domestic investments in the sector 1 (buildings and locally produced investment goods) and τ_{m}^i the effective tax rate on imported investments goods (including tariffs among others). This gives once log linearized:

$$q_{v,t}^j = \alpha_v^j(q_t^\star + \Delta\tau_m^i) + \left(1-\alpha_v^j\right)\Delta\tau_1^i \qquad (38a)$$

The final investment price faced by firms is:

$$q_{i,t}^j = \alpha_i^j q_{i,t}^v + \left(1-\alpha_i^j\right)q_{2,t} \qquad (38b)$$

The demand of the three types of investment in the three sectors j is given as follows, with τ_2^i being the tax rate on domestic investments in the sector 2.

$$inv_{2,t}^j = inv_t^j + \eta_i\left(q_{i,t}-q_{2,t}-\Delta\tau_2^{\mathrm{i}}\right) \qquad (39a)$$

$$inv_{1,t}^j = inv_t^j + \eta_i\left(q_{i,t}^j-\Delta\tau_1^{\mathrm{i}}\right) + (\eta_v-\eta_i)q_{v,t}^j \qquad (39b)$$

$$m_t^j = inv_t^j + \eta_i\left(q_{i,t}^j-\Delta\tau_1^{\mathrm{i}}\right) + +(\eta_v-\eta_i)q_{v,t}^j - \eta_v(q_t^\star+\Delta\tau_m^i) \qquad (39c)$$

The public sector

Government spending and the fiscal rule

The government raises taxes on profit (through the corporate tax), capital income, consumption, and labour income. The government produces public services such as administrative, health and education services using labour and capital. Public capital is exogenous, but public labour is endogenously determined given the stock of public capital and the level of public consumption. This public consumption ($g_{3,t}$ in log) is endogenous and set by a fiscal rule, to ensure the stability of the public debt over GDP ratio in the long run. The value of the long-term level of public consumption is also endogenous (see below).

$$g_{3,t} = \rho_g g_{3,t-1} + (1-\rho_g)\overline{g_3} - \eta_g(det_{-1} - \overline{det}) \quad (40)$$

The government makes monetary transfers (tr_t in log) – through child benefits and pensions – to households. These transfers are endogenously adjusted to ensure consistency in equilibrium. Prior to the pandemic, there were no transfers to unemployed working-age individuals, thus transfers are not assumed to impact labour participation or the reservation wage. The government also builds and maintains public infrastructure. We assume that maintenance spending preserves the duration of infrastructure. According to South Africa National Road Authority (2016), preventive maintenance every 6 years (resealing) costs one-sixth times major repairs every 12 years (pothole filling) and one-eighteenth times fully rebuilding the road every 16 years.[5] After factoring in the frequency of operations, preventive maintenance is still three times as cheap as major maintenance and about 7 times cheaper than waiting for the road to be impracticable to repair it. Obviously, capital cannot be added by preventive maintenance spending, so there is a maximum level of spending possible, which is about $prev^\star = \frac{2}{45}$ the value[6] of the infrastructure to be spent every six years, or equivalently 0.75% every year. We can then describe the preventive maintenance policy with a new variable $prev \in [0,1]$ and a parameter $\kappa_3 \approx 6.75 \times 0.75\% = 5\%$. So preventive maintenance allows to reduce the depreciation rate by 5%, down from 6.9% without intervention. The capital infrastructure accumulation equation becomes (in level):

$$K_{3,t} = (1 - \delta_3 + \kappa_3 prev_t)K_{3,t-1} + Inv_{3,t} \quad (41)$$

This equation can be log-linearised, assuming that in South Africa at the current steady state, there is no preventive maintenance. One should note that the steady state depends on the long-term policy of preventive maintenance captured by $\Delta prev$.

$$k_{3,t} = \left(\frac{1-\delta_3+\kappa_3\Delta prev^*}{1+\Gamma}\right)k_{3,t-1} + \left(1 - \frac{1-\delta_3+\eta\kappa_3\Delta prev_t}{1+\Gamma}\right)inv_{3,t} + \kappa(\Delta prev_t - \Delta prev^*) \quad (42)$$

The fiscal deficit DEF can be written as a function of the public consumption G_3, public investment, preventive maintenance, transfers TR, tax TAX and public debt interests which are endogenously determined by tax rates:

$$DEF_t = P_{3,t}(G_{3,t} + INV_{3,t}) + P_{3,t}prev^\star prev_t K_{3,t-1} + TR_t - TAX_t + i_{t-1}DET_{-1} \quad (43a)$$

This equation can be divided by the GDP deflator to obtain ratios, namely $def_t = \frac{DEF_t}{H_t}$, $det_t = \frac{DET_t}{H_t}$: In the long-run, the deviation in the debt ratio vis a vis the steady growth path will eventually match the deviation in the deficit expressed as a ratio of GDP, see equation (44c). This allows rewriting equation (43a) as:

$$det_t H_t = \frac{P_{3,t}}{P_t^H}(G_{3,t} + INV_{3,t}) + prev^\star prev_t \frac{K_{3,t-1}}{H_t}H_t + TR_t - TAX_t + i_{t-1}det_{t-1}H_{t-1}\frac{P_{t-1}^H}{P_t^H} \quad (43b)$$

This equation can be linearised, with φ_3^t, φ_3^x, φ_3^g, φ_3^i, φ_3^r and φ_3^d, denoting the shares of respectively transfers, taxes, public consumption, public investment, public interest payments and public deficit in nominal GDP at the steady state:

[5] It is to be noted that 16 years is not the total duration of the road infrastructure since it needs to be rebuilt when it becomes impracticable, which happens at around 80% of its lifetime, or 20 years.

[6] Or 1/18 of the value needed for reconstruction, which is four-fifths of the total value of the infrastructure.

$$\varphi_3^d(\Delta def_t + h_t) + \varphi_3^x tax_t = \varphi_3^t tr_t + \varphi_3^g(q_{3.t} + g_{3,t}) + \varphi_3^i(q_{3.t} + inv_{3,t}) + \frac{prev^\star}{1+\Gamma}\frac{K_3^*}{H^*}(q_{3.t} + \Delta prev_t + prev^* k_{3,t-1}) + \varphi_3^r(\Delta i_{t-1} + det_{-1} + h_{t-1} - \pi_t) \quad (43c)$$

Because the deficit includes the debt interests, the dynamics of public debt (in level) is simply:

$$DET_t = DET_{t-1} + DEF_t \quad (44a)$$

So, the dynamics of the debt-to-GDP ratio becomes:

$$\frac{DET_t}{H_t} = \frac{DET_{t-1}}{H_{t-1}}\frac{Y_{t-1}}{Y_t}\frac{P_{t-1}^H}{P_t^H} + \frac{DEF_t}{H_t} \quad (44b)$$

This can be log-linearised around the steady state:

$$\Delta det_t = \frac{1}{\Gamma\pi^*}(\Delta det_{t-1} + y_{t-1} - y_t - \pi_t) + \left(1 - \frac{1}{\Gamma\pi^*}\right)\Delta def_t \quad (44c)$$

It follows from this equation that in the long run, the deviation in the debt ratio ($\overline{det}$) equals the deviation in the deficit ($\overline{def}$).

The target public consumption level $\overline{g_3}$ is computed as a function of the tax rates and the level of investment chosen so that at the steady state, public debt-to-GDP ratio would match its targeted level. To compute this value, we start from the definition of the public deficit (directly using the log-linearised version) and we take the long-term values (we approximate long-term levels by values taken at the 100th period):

$$\varphi_3^d(\overline{det} + y_{100}) + \varphi_3^x tax_{100} = \varphi_3^t\, \overline{tr} + \varphi_3^g(q_{3,100} + \overline{g_3}) + \varphi_3^i(q_{3,100} + \overline{inv_3}) + \frac{prev^\star}{1+\Gamma}\frac{K_3^*}{H^*} prev^* \overline{k_3} + \varphi_3^r(\overline{det} + h_{100}) \quad (45)$$

Government financing

The total fiscal revenues can be expressed as follows in a log-linearized fashion, with φ_3^{xk} the share in GDP of each type of taxes among production, consumption, capital, and labour.

$$\varphi_3^x tax_t = \sum_{k\in\{C,K,L\}} \varphi_3^{xk} tax_t^k \quad (46)$$

Each type of tax is deducted from tax rates definitions. Taxes on capital are paid on capital income and on profits:

$$TAX_t^K = \tau_t^K\left(R_{t-1}(K_{1,t} + K_{2,t}) + \frac{1}{\theta_{1,t} - 1}Y_{1,t}(1 - \psi_1 Q_{Z,t}^1) + \frac{1}{\theta_{2,t} - 1}Y_{2,t}(Q_{2,t} - \psi_2 Q_{Z,t}^2)\right) \quad (47a)$$

This can also be log-linearised as follows:

$$
\begin{aligned}
tax_t^K = \frac{\tau^K}{\tau^{K*}} - 1 \\
&+ \frac{1}{T_K}\left\{\left((K_{1*} + K_{2*})r_{t-1} + K_{1*}k_{1,t-1} + K_{2*}k_{2,t-1}\right)\right. \\
&+ \frac{Y_{1*}(1-\psi_1)}{\theta_{1*}-1}\left(y_{1,t} - \frac{\psi_1}{1-\psi_1}q_{1,t}^Z - \left(\frac{\theta_{1,t}-\theta_{1,*}}{\theta_{1,*}-1}\right)\right) \\
&\left.+ \frac{Y_{2*}(1-\psi_2)}{\theta_{2*}-1}\left(y_{2,t} + \frac{q_{2,t}-\psi_2 q_{2,t}^Z}{1-\psi_2} - \left(\frac{\theta_{2,t}-\theta_{2,*}}{\theta_{2,*}-1}\right)\right)\right\} \quad (47b)
\end{aligned}
$$

The factor T_K is the value of capital income at the steady state:

$$
T_K = R^*(K_{1*} + K_{2*}) + \frac{Y_{1*}(1-\psi_1)}{\theta_{1*}-1} + \frac{Y_{2*}(1-\psi_2)}{\theta_{2*}-1} \quad (48)
$$

Taxes on labour depend on the share of labour income earned in each market $a_x^L, x \in \{A, B\}$ by unconstrained households and the share of labour income earned in each market by Ricardian agents $a_x^o, x \in \{A, B, C\}$ as labour tax rates differ from both types of consumers.

$$
tax_t^L = a_A^L\left(l_t^A + w_t^A + a_A^o\left(\frac{\tau_t^{L0}}{\tau_*^{L0}} - 1\right) + (1-a_A^o)\left(\frac{\tau_t^{Lc}}{\tau_*^{Lc}} - 1\right)\right) + a_A^L\left(l_t^B + w_t^B + a_B^o\left(\frac{\tau_t^{L0}}{\tau_*^{L0}} - 1\right) + (1-a_B^o)\left(\frac{\tau_t^{Lc}}{\tau_*^{Lc}} - 1\right)\right) + (1 - a_A^L - a_B^L)\left(l_t^C + w_t^C + a_C^o\left(\frac{\tau_t^{L0}}{\tau_*^{L0}} - 1\right) + (1-a_C^o)\left(\frac{\tau_t^{Lc}}{\tau_*^{Lc}} - 1\right)\right) \quad (49)
$$

Taxes on consumption depends on the tax rate on import τ_t^m and on domestic consumption τ_t^c and the shares of consumption by products and types of consumers:

$$
\begin{aligned}
tax_t^C = a_c^m\left(\left(\frac{\tau_t^m}{\tau_*^m} - 1\right) + \widehat{q_t} + a_c^o m_{c,t}^c + (1-a_c^o)m_{c,t}^o\right) \\
&+ (1-a_c^m)\left(\left(\frac{\tau_t^C}{\tau_*^C} - 1\right)\right. \\
&\left.+ \frac{dc_{1*}^o dc_{1,t}^o + dc_{1*}^c dc_{1,t}^c + q_*^2\left(dc_{2*}^o dc_{2,t}^c + dc_{2*}^c dc_{2,t}^o\right) + q_{2,t}(dc_{2*}^o + dc_{2*}^c)}{dc_{1*}^o + dc_{1*}^c + q_*^2(dc_{2*}^o d + dc_{2*}^c)}\right) \quad (50)
\end{aligned}
$$

Public investment and total factor productivity

We assume that the stock of infrastructure – road and rail networks, ports, communication networks and electricity generation – has a direct impact on TFP. Another way to look at this effect would be to consider that additional public capital stock is boosting factor productivity of private capital and labour. We rather use the "TFP" angle here because the public capital stock cannot be measured precisely and its decline through inefficient investments and poor maintenance have not been captured in national accounts. Thus, an observed decline of the TFP which our model is modelling as a (partial) consequence of declining capital stock.

$$
A_t = A_t^o\left(\frac{K_{3,t}}{\frac{H_t}{P_t^H}}\right)^{\Xi} \Rightarrow a_t = a_t^0 + \Xi\left(k_{3,t} - h_t + p_t^H\right) \quad (51)
$$

In the long run, the growth rate of the public capital stock equals the growth rate of the economy (Γ) and the growth rate of maintenance spending.

$$\frac{K_3^*}{H^*} = \frac{1+\Gamma}{\Gamma + \delta_3 - \eta_3 \Delta prev^*} \left(\frac{INV_3^*}{\frac{H^*}{P^H}} \right) \quad (52)$$

Monetary policy

The dynamics of foreign and domestic prices depend on the monetary policies set domestically and abroad. The domestic inflation rate is set following a Taylor rule, where gdp is the real GDP gap with the steady state in log:

$$i_t = \rho_i i_{t-1} + \varrho_P \pi_t + \varrho_Y (gdp_t - gdp_{t-1}) \quad (53)$$

The foreign interest rate is also set using a Taylor rule:

$$i_t^* = \rho_i^* i_{t-1}^* + \varrho_P^* \pi_t^* + \varepsilon_{i,t}^* \quad (54)$$

The domestic inflation rate can be deduced from the terms of trade and the foreign inflation rate, using the definition of the terms of trade[7]. Foreign inflation remains exogenous in this small open economy, allowing to compute the change in the terms of trades from the changes in domestic inflation.

$$\pi_t = \pi_t^* - q_t^\star + q_{t-1}^\star + e_t - e_{t-1} \quad (55)$$

Inflation in sector 2 can also be deduced from the general inflation and the relative price of goods and services in sector 2:

$$\pi_t^2 = \pi_t + q_{2,t} - q_{2,t-1} \quad (56)$$

Shocks

The economy is exposed to a series of exogenous shocks. The foreign inflation rate is following the dynamics below:

$$\pi_t^* = \rho_\pi \pi_t^* + \varepsilon_t^\pi \quad (57)$$

The real growth rate of GDP (the sum of total factor productivity growth and the growth of the labour force) is following a similar dynamic:

$$\Gamma_t = \rho_\Gamma \Gamma_{t-1} + \varepsilon_t^\Gamma \quad (58)$$

The productivity in sector 1 is following a similar dynamic:

$$A_{1,t} = \rho_{1,A} A_{1,t-1} + \varepsilon_t^{A1} \quad (59)$$

Equilibrium

The balance between supply and demand is written in the three product markets. In the tradable sector, it follows (in levels), with Q_x the relative export price (terms of trades),

$$Y_1 = \sum_j Z_1^j + Inv_1^j + C_1^o + C_1^c + \bar{X}(Q_x)^{-\eta_x} \quad (60)$$

The equilibrium in the non-tradable sector is similar, but without any exports:

$$Y_2 = \sum_j Z_2^j + Inv_2^j + C_2^o + C_2^c \quad (61)$$

[7] See appendix for derivation

In the public sector, supply goes to public consumption.

$$Y_3 = G_3 \qquad (62)$$

The labour market balances the aggregate supply and the demand from each sector for both high-skilled and low-skilled labour separately. High-skilled workers can be hired from each sector:

$$L^A = L_1^A + L_2^A + L_3^A \qquad (63)$$

In the submarket B, low-skilled workers can be hired from the public sector or the tradable sector.

$$L^B = L_1^B + L_3^B \qquad (64)$$

The demand of labour in submarket C equals the demand of low-skilled labour in the non-tradable sector by definition. The equilibrium on foreign goods has been described above.

Income and wealth distributions

Income and wealth inequalities are large in South Africa, therefore fiscal reforms and macroeconomic shocks are likely to have different impacts on the distinct segments of the population. We cluster agents in income groups *k* which we order by per capita income.

Consumption distribution

Consumption derives either from disposable income (for liquidity-constrained agents) or from expected income (for Ricardian agents). We also assume that the share of each type of consumers in a group is unaffected by the shocks simulated. We have already introduced above the shares of labour income in each submarket going to either Ricardian or constrained consumers. We can define the shares of the consumption in each group for Ricardian consumers denoted $\aleph^k$.

$$\aleph^k = \frac{C_*^o}{C_*^k} \qquad (65)$$

This allows log-linearising the consumption for each group, denoting y_t^k the income of agents in group *k.* In practice however, the coefficient $\aleph^k$ are trivial for all deciles but the 9th, as people below the 80th percentile are all considered to be liquidity constrained while people in the 10th decile are all assumed to be Ricardian. In the 9th decile, about half of agents are assumed to be constrained.

$$c_t^k = \left(1 - \aleph^k\right) y_t^k + \aleph^k\, c_t^o \qquad (66)$$

Our approach could appear restrictive since we do not allow people to move from one decile to another. The degree of income inequality would however not be affected by allowing people to move between deciles because the weight of a decile is by definition set to 1/10 and so any measure of inequality is entirely derived from the distribution of income in each decile, once we admit of course that the problem can be correctly described by restraining the full income distribution to the 10 deciles. Mobility would however modify the inequality in wealth over time because if people are more mobile between deciles differences in dynasties' wealth would be lessened. But given that wealth inequalities are so skewed currently, any limited policy changes or macro-shocks that the model is considering would not realistically impact this wealth distribution. What we are capturing here are the differentiated impacts of policy/shocks on the different deciles, which is not only illustrative but allows quantifying the distributional effects of policies. The model does not permit however for an endogenous modification of the share of liquidity-constrained households, which remains exogenous. That is not possible in a DSGE perspective, because assuming that liquidity constraints could be endogenous would require to track the wealth of a very large number of heterogeneous agents, which is impracticable. Moreover, the use of heterogeneous agents is a necessary condition for endogenous liquidity constraints but unfortunately not sufficient. There have been recent attempts to introduce Heterogeneous Agents Neo-Keynesian (so called "HANK) models by simulating the distribution. But the benefits of such an approach are not clear in face of the complexity introduced and they do not solve the issue of endogenous liquidity as the share of liquidity-constrained agents remain exogenous. This issue although of a theoretical interest is not critical in the case of South Africa, because of the income and wealth gap that split the society in two groups.

Income distribution

By mapping the different types of workers and consumers to the income groups, one can track the evolution of income, consumption, and wealth on average in each group. The income of the group k, Y^k can be related to employment $L_{i,t}^k$ in the group in each submarket, the wage in the three submarkets, tax rates $\tau_{L,t}^k$, transfers and capital $K_{j,t}^k$. The parameter $\upsilon_{P,t}^{j,k}$ is the share of the capital of firms held by people in the group k.

$$Y_t^k = \sum_{i=\{A,B,C\}} L_{i,t}^k W_{i,t}\left(1-\tau_{L,t}^k\right) + TR_t^k + \sum_{j=\{1,2\}} R_t K_{j,t}^k \left(1-\tau_t^K\right) + \sum_{j=\{1,2\}} \upsilon_{P,t}^{j,k} \frac{1}{\theta_t^j} P_t^j Y_t^j \left(1-\tau_{P,t}\right) \quad (67a)$$

These equations can be log-linearized around the steady state:

$$\begin{aligned} y_t^k = & \sum_{i=\{A,B,C\}} \frac{L_{i*}^k W_{i*}\left(1-\tau_{L,*}^k\right)}{Y_*^j}\left(w_{i,t} + l_{i,t}^k - \frac{\Delta\tau_{L,t}^k}{1-\tau_{L,*}^k}\right) + \frac{TR_*^k}{Y_*^k} tr_t^k \\ & + \sum_{j=\{1,2\}} \frac{R_* K_{j*}^k}{Y_*^j}\left(r_t + k_{j,t}^k - \frac{\Delta\tau_{K,t}}{1-\tau_{K*}}\right) \\ & + \sum_{j=\{1,2\}} \upsilon_{P*}^{j,k} \frac{P_*^j Y_*^j}{\theta_*^j Y_*^j}\left(1-\tau_{P*}\right)\left(-\frac{\theta_{i,t}-\theta_i^*}{\theta_i^*} + y_{j,t} + q_{j,t} + \frac{\Delta\upsilon_{P,t}^{j,k}}{\upsilon_{P*}^{j,k}} - \frac{\Delta\tau_{P,t}}{1-\tau_{P*}}\right) \quad (67b) \end{aligned}$$

Equations (67b) can be evaluated by making additional simplifying assumptions. First, we assume that the change in labour is homogeneous in all groups of workers. This assumption is natural since we only consider three types of workers.

$$l_{i,t}^k = l_{i,t} \quad (68a)$$

A related assumption is that the change in the productive capital in each sector is also homogeneous among income groups. This approximation is less obvious, but it does not bear any major bias to our results since people with liquidity constrains, that is the poorest 85% of the population, do not hold meaningful amount of productive capital. Most of their capital is housing, whose related income does vary with the overall cost of capital.

$$k_{j,t}^k = k_{j,t} \ (68b)$$

A third assumption concerns the way transfers are operated. We assume for simplicity in the standard variants that the transfers are increased in proportion of their distribution at the steady state. That would imply an increase in the amounts transferred rather than the introduction of new (and more targeted benefits). The model would allow in practice to differentiate benefits by deciles. So, in the standard transfers variants one has:

$$tr_t^k = tr_t \quad (69)$$

The effective tax rate on labour income depends on the level of income. We assume that people who are not constrained (the poorest 85%) are paying nothing while the other are paying a tax $\tau_{L,*}^o$.

Finally, because we cannot model the change in liquidity constrains, these constraints are assumed to be exogenous. Therefore, there is no reason to believe that liquidity-constrained agents would be allowed to buy stocks or corporate shares. As a consequence, the share of firms capital held by the different groups, the $\upsilon_{P,t}^{j,k}$ remain exogenous and one has:

$$\frac{\Delta\upsilon_{P,t}^{j,k}}{\upsilon_{P*}^{j,k}} = 0$$

To calibrate equation (68b) and set each type of income at the steady state, we use analysis of the National Income Dynamics Study (NDIS) survey.

Wealth distribution

Income inequalities translate into wealth inequalities, which determines future streams of capital income. There are two types of capital investments in the model, productive capital, which is lent at the cost of capital R to businesses and equity, which is remunerated by profits deriving from imperfect competition and the ownership of firms. These capitals are distributed between groups by looking at the net financial wealth and net business assets of each decile. We know that the data from the latest NDIS survey cannot be used at face value because the sampling is skewed, leading to large underestimation of income, especially at the higher end. We then draw the distribution of assets from the synthesis undertaken by Chatterjee et al. (2020) by mixing not only survey data but also tax and central bank data. They report that the bottom 50% of the population has a negative wealth, while the middle 40% embedding deciles 6 to 9 owns 17% of the national wealth. The top 10%, the first decile, owns 86% of national wealth, but the top 1% captures 55%, the top 0.1%, about 30% and the top 0.01% about 15%. Wealth can be split according to the type of asset, see Table 1.

Table 1: Distribution of assets in billion Rands in 2017, by types of assets

Group	Percentiles	Wealth	Currency	Business assets	Housing	Pensions/life insurance	Bonds & stocks
Bottom_50	0-50	-283.2	6.7	5.8	465.6	198.9	0.0
Middle_40	50-90	1,954.1	19.2	162.5	904.6	1,159.7	8.0
Lower_top	90-99	3,564.8	36.1	73.6	1,031.0	1,865.2	183.8
Top_1	99-99.99	4,590.7	6.3	118.5	641.9	450.4	1,298.5
Higher_end	99.9-100	1,721.2	1.0	55.7	282.7	78.8	2,505.1
Total	0-100	11,547.5	69.3	415.7	3,325.7	3,752.9	3,995.4

Source: Chatterjee et al. (2020)

Wealth evolves in two ways: (i) through the accumulation of savings and (ii) through changes in the prices of assets. As the model is already tracking consumption and income, we can deduce total savings by income groups. The distribution of savings between different assets, namely domestic and foreign bonds and physical capital is not tracked in a typical DSGE model, where everything is determined through an arbitrage condition between the returns of different assets. In the framework we have chosen, there is no external position so that the movements in the stock of foreign bonds, which are described by the model are limited. In a typical DSGE, the quantity of private bonds is usually null at the aggregate level as the debt issued by lenders is accrued by borrowers. There can be a positive stock of government bonds, which is mirroring the fiscal balance, but there is no net creation of money in a basic DSGE to finance private capital and all capital accumulation flows through variations in the capital stock. The increase in government bond is not adding wealth to optimizing agents precisely because of the Ricardian equivalence, that this accrued debt must be paid off eventually by higher taxes in the future.

To stay in the spirit of such a modelling, we assume that thanks to the arbitrage condition, all wealth variations can be captured through changes in the physical capital stock. Because competition is not perfect though, producers make profit and net wealth derives from lending the capital to firms and capturing profits. The value of stocks is assumed to equal the discounted sum of future profits. This gives the following definition for net wealth (NW) in level, with S_t^j the share of stocks owned by the group *j* and P_t^S the price of stocks:

$$NW_t^j = K_t^j + S_t^j P_t^S \quad (70)$$

The price of stock (relative to the production price in sector 1) P_t^S is equal to the expected discounted sum of future profits. Because the profits are uncertain, it is necessary to take into account a risk premium, r^p, which we assume exogenous for simplicity:

$$\frac{{P_t}^S}{P_t} = \sum_{s=t}^{\infty}\left(\prod_{u=s}^{t}\frac{1+\pi_u}{1+i_u+r_u^p}\right)(1+\Gamma)^{t-s}\sum_{i=\{1,2\}}\frac{Q_s^i Y_s^i}{\theta_s^i}(1-\tau_s^P) \qquad (71)$$

This equation can be log-linearised at the steady state by denoting the share of profits coming from the sector i as

$$\varsigma_i = \frac{\frac{Q_*^i Y_*^i}{\theta_*^i}}{\frac{Q_*^i Y_*^i}{\theta_*^i} + \frac{Q_*^{-i} Y_*^{-i}}{\theta_*^{-i}}} \quad (72)$$

Recalling the value of the nominal interest rate at the steady state, and using first-order approximation, it follows:

$$\frac{1+\pi_*}{1+i_*} = \frac{\beta}{1+\Gamma} \Rightarrow \frac{(1+\pi_*)(1+\Gamma)}{(1+i_*+r^p)} \approx \frac{\beta}{1+r^p}$$

This allows determining the evolution of the stock market price around the steady state:

$$p_t^S = \left(1-\frac{\beta}{1+r^p}\right)\sum_{s=t}^{\infty}\left(\frac{\beta}{1+r^p}\right)^s\left\{\sum_{i=\{1,2\}}\varsigma_i\left(q_s^i + y_s^i - \frac{\theta_{i,s}-\theta_i^*}{\theta_i^*} - \frac{\Delta\tau_s^P}{1-\tau_*^P}\right) + \sum_{u=s}^{t}\pi_s - \frac{\Delta i_u}{1+i^*}\right\} \quad (73)$$

The last term which captures the short-term variations of the wealth price can be neglected before the variations in volumes. Finally, the evolution of net wealth can be log-linearised, after introducing the relative weight of stocks in total wealth of the group *j*, ς_S^j

$$nw_t^j = (1-\varsigma_S^j)k_t^j + \varsigma_S^j p_t^S \qquad (74)$$

The capital stock of the group follows the evolution of the disposable net income $\widehat{y_t^j}$ and consumption $\widehat{c_t^j}$

$$k_t^j = \left(\frac{1-\delta^j}{1+\Gamma}\right)k_{t-1}^j + \left(1-\frac{1-\delta^j}{1+\Gamma}\right)\left(\left(\frac{Y_*^j}{Y_*^j - C_*^j}\right)y_t^j - \left(\frac{C_*^j}{Y_*^j - C_*^j}\right)c_t^j\right) \quad (75)$$

Calibration

The structure of the economy

The structure of the sectors in the model is based on the National accounts as displayed in Table 2. The agriculture sector is not included, as South Africa only exports a tiny fraction of its agriculture. Trade and transportation are, however, included in the tradable sector as exports of mining and manufacturing embed a large share of transportation and trade services. The community, social and personal services sector has been split according to detailed industry in order to separate the personal services (such as domestic services) from public services as employment tend to be highly skilled and wages tend to be quite elevated in the public sector while private sector workers are on the contrary rather low-educated and poorly paid.

Table 2. Matching the model's sector with national accounts' industries

Industry	Detailed industry	Sector 1 (tradable)	Sector 2 (non-tradable)	Sector 3 (public)
Agriculture			X	
Mining		X		
Manufacturing		X		
Utilities			X	
Construction			X	
Trade		X		
Transport		X		
Business services			X	
Community, social and personal services	Education			X
	Health and social work			X
	Other community activities			X
	Personal services		X	

The producers are assumed to evolve in an imperfect monopolistic competition environment, where prices are basically set by applying a fixed margin rate on the marginal cost.[8] As mark-ups are very heterogeneous between industries in South Africa, the average mark-ups in the two aggregate sectors of the model do not necessarily reflect this complexity. For simplicity, it is assumed that the public sector does not make profit, which is not far from reality. While total employment is quite balanced between the tradable and non-tradable sectors, the average wage is about twice as large in the tradable sector because the productivity is higher, the sector is overall more competitive and less intensive in capital. In contrast, the business service sector is very intensive in real estate capital. The main characteristics of the sectors are displayed in Table 3.

Table 3. Main characteristics of sectors in 2015

Sector	Value-added	Output	Employment	Gross monthly wage	Mark-up	Investment
1: Tradable	1,714	4,758	6,361	11,416	1.130	271
2: Non-tradable	1,239	2,338	6,485	6,220	1.164	380
3: Public services	727	1,058	3,082	13,819	1.000	132
Total economy	3,680	8,154	15,928	31,454	-	783

Note: Value-added, output and investments are displayed in million Rand 2015. Gross monthly wages are in Rand 2015, Employment figures are in thousands.
Source: National accounts; and authors' calculations from the Quarterly Labour Force survey, QLFS (2015).

[8] The marginal cost in the model accounts for labour and capital but also intermediate inputs. Therefore, the apparent elasticity of substitution between products tend to be higher than when the cost of intermediate inputs is not taken account.

Features of the labour market

To gauge the potential effects of wage policies on the economy, the sector considers two types of workers and three distinct labour markets. Workers who completed secondary education are considered as high-skilled, while the rest of the active population is assumed to be low-skilled. High-skilled workers are assumed to be hired from a closed labour market denoted A. Low-skilled workers can work either in the submarket B which gathers hirings from the tradable sector and the public sector and the submarket C, which is related to the non-tradable sector. Average wages differ substantially from the three submarkets, as can be seen in Table 4.

Table 4. Features of the labour markets

Submarket	Gross monthly Wage (2015)	Total employment (2015)	Share of the labour force	Share of the total labour compensation
A	15,322	8,167,905	51.3%	80.5%
B	5,794	3,955,962	24.8%	14.7%
C	1,965	3,804,485	23.9%	4.8%
Total	9,765	15,928,352		

Source: Authors' calculations from the QLFS (2015)

The characteristics of the bargaining processes in the submarkets are displayed in Table 5. The differences between the remuneration of the low-skilled workers in the tradable and non-tradable sectors are governed by lower reservation wage linked to the differentiation in the coverage of unemployment benefits and the lower bargaining power – linked to high informality among service workers – rather than the differentiation between the firing or hiring probabilities.[9,10]

Table 5. Features of the wage-bargaining process in the three submarkets

Submarket	Workers' bargaining power	Reservation wage	Firing probability	Hiring probability
A: High-skilled	64.5%	12,135	19.0%	41.6%
B: Low-skilled in tradable	74.4%	3,501	24.0%	18.9%
C: Low-skilled in non-tradable	16.6%	526	24.0%	18.1%

Source: Authors' calculations from the LFS (2013/2014) and National Accounts.

Calibration of the wealth distribution

The poorest half of the population has no wealth at all, so they are fully liquidity-constrained (Chaterjee, 2020). In addition, up to the 70th percentile, households do not possess any bonds or stocks, which are somewhat liquid, but only illiquid assets such as housing, life insurance and pensions funds or business assets. Households between the 70th and 80th percentiles own about 2% of their assets in tenant-occupied housing, which represents 3.7% of their pre-tax income. As such assets are not fully liquid, we still consider that they are liquidity-constrained. Moreover, their ratios debt-to-pre-tax income ratio and debt-to-net wealth is half of that for households above the ninth decile, indicating that they are also liquidity-constrained.

[9] The firing probabilities are calibrated using the transition matrix between sectors or in and out of employment from one quarter to another, based on the dedicated LFS survey with panel data 2013Q4-2014Q1.

[10] The hiring probabilities are built using the overall unemployment rate for low-skilled workers and hirings in the submarkets based on firing probabilities.

People between the 80th and 90th percentiles tend to own stocks and bonds, which are liquid and their real estate investments on top of their own dwellings represent about 6.5% of their net wealth. In addition, about 40% of people in the 9th decile have medical coverage. Therefore, it is assumed that about half of households in this decile are liquidity-constrained. Finally, households in the 90th percentile and above are assumed to be Ricardian, as they own a sizeable portfolio of liquid assets.

For liquidity-constrained households, consumption evolves in line with the disposable income, which would allow some of them to accrue wealth because they consume a fixed share of their income. This share can be calibrated to match their 2017 wealth statistics. At the steady state, we can use the accumulation equation of capital to link the capital stock with the savings rate:

$$\frac{Y_*^j - C_*^j}{Y_*^j} = \frac{K_*^j}{Y_*^j}\left(1 - \frac{1-\delta^j}{1+\Gamma}\right)$$

The depreciation rates of real estate and business assets are taken to be 3 and 7% respectively, using the relative weight of each asset stock. This rate is used for financial assets, assuming that an arbitrage condition holds. The long-term productivity growth rate is taken as 2%. This leads to a savings ratio above 100% for agents in the top 0.1 percentile. The level of consumption in this group is assumed to equal the level of consumption in the top percentile to deduce the savings rate to be used to describe the dynamics of the capital stock. Using average pre-tax income and net wealth by percentiles from Chaterjee et al. (2020) leads to the following fixed savings rates for liquidity-constrained agents (Table 6).

Table 6. Savings rate of liquidity-constrained agents by decile

Income group	Real estate	Business capital	Equity	Depreciation rate	Net wealth-to-income	Savings rate	The share of the liquidity constrained	Wealth share
	----- As a share of assets -----							
Decile 1	0.0%	0%	0	7.0%	-22.77	0.0%	100%	0.0%
Decile 2	31.6%	1%	0	5.7%	-0.14	0.0%	100%	0.0%
Decile 3	40.6%	2%	0	5.4%	0.10	0.7%	100%	0.0%
Decile 4	46.5%	3%	0	5.1%	0.55	3.8%	100%	0.3%
Decile 5	53.5%	3%	0	4.9%	0.85	5.7%	100%	0.6%
Decile 6	56.8%	8%	0	4.7%	1.07	7.0%	100%	1.1%
Decile 7	47.7%	7%	0	5.1%	1.32	9.2%	100%	2.0%
Decile 8	41.3%	6%	0	5.3%	1.63	11.8%	100%	3.9%
Decile 9	42.6%	8%	0	5.3%	1.97	14.1%	50%	9.3%
90-99%	38.7%	6%	0	5.5%	1.92	14.0%	0%	29.8%
99-99.9%	36.8%	5%	34%	5.5%	5.29	39.0%	0%	28.8%
top 0.1%	13.5%	1%	55%	4.0%	14.78	80%	0%	24.2%

Source: Authors' calculations from Chaterjee et al. (2020).

Calibration of the remaining parameters

The model is log-linearised around its steady state to allow numerical approximation. Long run targeted moments in the model are determined by accounting ratios derived from the national accounts, as well as price elasticities, taking past estimates for some and calibrating the others (Table 7).

Table 7. Values of price elasticities in the model

Elasticities	Value	Type
Capital labour	1	Calibrated
High/low skill labour	0.5	Calibrated
Export (sector 1)	1.4	Literature
Investment import	0.9	Estimated
Investment tradable/non-tradable	0.6	Calibrated
Consumption import	0.9	Estimated
Consumption tradable/non-tradable	0.7	Calibrated
Intermediate input import	0.9	Estimated
Intermediate input tradable/non-tradable	0.6	Calibrated
High-skilled labour participation	0.4	Estimated
Low-skilled labour participation	0.6	Estimated

Note: Edwards and Lawrence (2006) found the exports price elasticity to be between 1.3 and 1.6. Golub and Ceglowski (2002) find even smaller exports price elasticities, between -0.8 to -1.4. A naive estimation of the elasticity gives 0.9, which confirms that exports in South Africa do not respond as much as to changes in competitiveness as other countries.

Short-term responses of the model depend on three types of rigidities: (i) price rigidities, modelled with Calvo-type pricing[11] in both private and public sectors, export prices and wages; (ii) Monetary policy as described by an estimated Taylor-rule for South Africa and (iii) fiscal policy, where both public spending on goods and transfers are assumed to react to the level of public debt in the long run, ensuring that the public debt-to-GDP ratio remains constant at the steady state.

Parameters that affect the short-term dynamics have been estimated using a Bayesian approach using the software Dynare[12], focusing on the quarterly data for South Africa between 1970 and 2016. The Bayesian estimates of model parameters are displayed in Table 8.

Table 8. Bayesian estimates of DSGE short-term parameters

Parameters	Prior mean	Posterior mean	90% lower interval	90% upper interval
Consumption habit	0.80	0.981	0.979	0.982
Capital adjustment cost	5.00	7.470	6.558	8.391
Spending lag	0.50	0.453	0.387	0.515
Spending reaction to debt ratio	0.50	0.004	0.003	0.006
Inflation lag: tradable	0.60	0.614	0.573	0.659
Calvo reset probabilities: tradable	0.50	0.799	0.784	0.811
Inflation lag: non-tradable	0.60	0.959	0.925	0.990
Calvo reset probabilities: non-tradable	0.50	0.692	0.675	0.706
Inflation lag: wages	0.50	0.635	0.566	0.700
Calvo reset probabilities: wages	0.50	0.896	0.891	0.901
Interest lag	0.50	0.979	0.978	0.980
Taylor coefficient: inflation	1.00	0.026	0.025	0.027
Taylor coefficient: growth	0.01	-1.062	-1.106	-1.024
Productivity lag in sector 1	0.90	0.838	0.822	0.855
TFP lag	0.90	0.997	0.995	0.998

[11] With Calvo pricing, producers can only set a fixed share of their price in each period. Moreover, some backward indexation can be introduced to reflect the idea that prices that cannot be reset to the optimal level may merely follow past inflation.

[12] Dynare version 4.4.3 for Mac OsX. The Metropolis-Hasting method with 20,000 replications.

Macroeconomic impacts of structural reforms

We investigate the consequences of structural reforms by studying the dynamic adjustment to market deregulation. Given the large size of the shocks, transition dynamics from the initial equilibrium to the final equilibrium are found by solving the deviation to the log-linearised steady state.

We model structural reforms as reductions in monopolistic mark-ups. In doing so, we follow much of the theoretical literature on the macroeconomic effects of product market reforms (e.g., Blanchard and Giavazzi, 2003; Eggertsson et al., 2014; Fernández-Villaverde et al., 2014, Andrés et al., 2017). An alternative line of research considers the effects of structural reforms in the context of frameworks with different product and labour market structures as in Cacciatore and Fiori (2016) and Cacciatore et al. (2016). In these papers, reforms are implemented in the form of reductions in producer entry costs, firing restrictions and unemployment benefits in models featuring endogenous producer entry and labour market frictions. The mark-ups approach eases the tractability of reforms in an already complex model and illustrates the competitiveness gains vis-à-vis the external sector and higher competition in the market. Both approaches are interchangeable as mark-ups are proportional to entry costs in the long run.

South Africa has a high level of product market regulation. The OECD 2018 Product Market Regulation (PMR) indicators show that the aggregate indicator for the economy is 2.53 for South Africa compared to 1.43 for the OECD average and 1.0 for the top 5 best performing countries. Therefore, we consider an arguably large reduction of 10% in the price mark-ups. This is, for instance, twice the 5% reduction in price mark-ups considered for Spain by Andrés et al. (2017), whilst the PMR for Spain is 1.03.

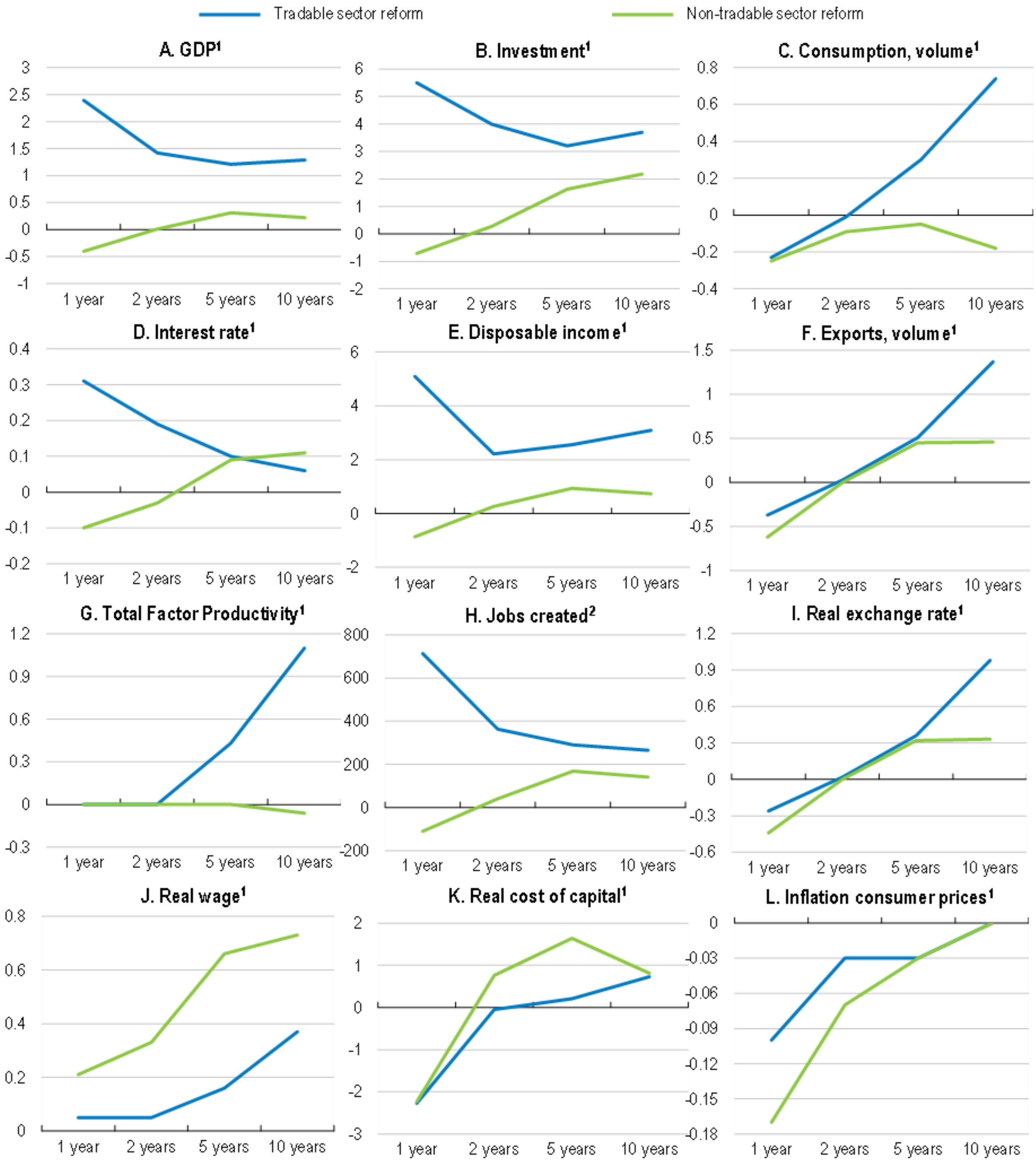
Regarding the impact of the product market reform, the two channels emphasised in the literature are the contractionary short-run substitution effect (Eggertsson et al., 2014; Cacciatore and Fiori, 2016 and Cacciatore et al., 2016), and the expansionary long run income effect (Fernández-Villaverde et al., 2014, Cacciatore and Fiori, 2016 and Cacciatore et al., 2016, Andrés et al., 2017). We show that deregulating product markets yields different short-term impacts in the tradable or non-tradable sectors, mainly due to the savings and investment responses of Ricardian households.

A 10% decline in the mark-up, $\left(\frac{\theta_1}{\theta_1 - 1}\right)$, in the tradable sector has a positive impact on GDP not only in the long run, but also in the short and medium-run relative to the initial steady state (blue line in Figure 2). In the long run, lower mark-ups increase external competitiveness, boosting exports, thus allowing to increase imports and domestic production and income. As Ricardian agents expect a strong increase in income in the future, they start investing rapidly to build up the required capital. This drives domestic investment and boosts GDP, following the reform. The rise in the labour demand additionally raises the disposable income of liquidity-constrained agents. Consumption is reduced slightly in the short run due to the tilting of domestic demand towards investment, but it increases in the long run, concurrent with higher wages and disposable income. Moreover, as GDP increases, government revenues grow, allowing larger public consumption and jobs, and distribution of income across the economy. Labour demand increases rapidly along with investment and recedes as the stock of private capital converges to its new steady state.

Enhancing competition on the non-tradable sector is recessionary in the short term (green line in Figure 6), as in the literature (Cacciatore and Fiori, 2016 and Cacciatore et al., 2016). Export costs are not directly lowered by this reform, nonetheless, exports still benefit from cheaper inputs and domestic capital in the long-run, as enhanced competition in the non-tradable sector reduces prices eventually. These effects are limited in size and gradual. As a result, Ricardian households ramp up their investment progressively. Falling prices in the short run reduce profits, which limit further investment because of capital adjustment costs. Labour demand also declines slightly. As consumer prices fall, because of rigidities in labour markets, real wage increases and imported consumption rises. The real exchange rate appreciates and exports decline, depressing aggregate demand in the short run. In the medium to long run, falling prices in the non-tradable sector starts benefitting exporters as domestic inputs and capital becomes cheaper.

However, the overall impact of the reform on GDP is modest in the long run, due to the smaller size of the non-tradable sector.

Figure 6. The impacts of product market reforms



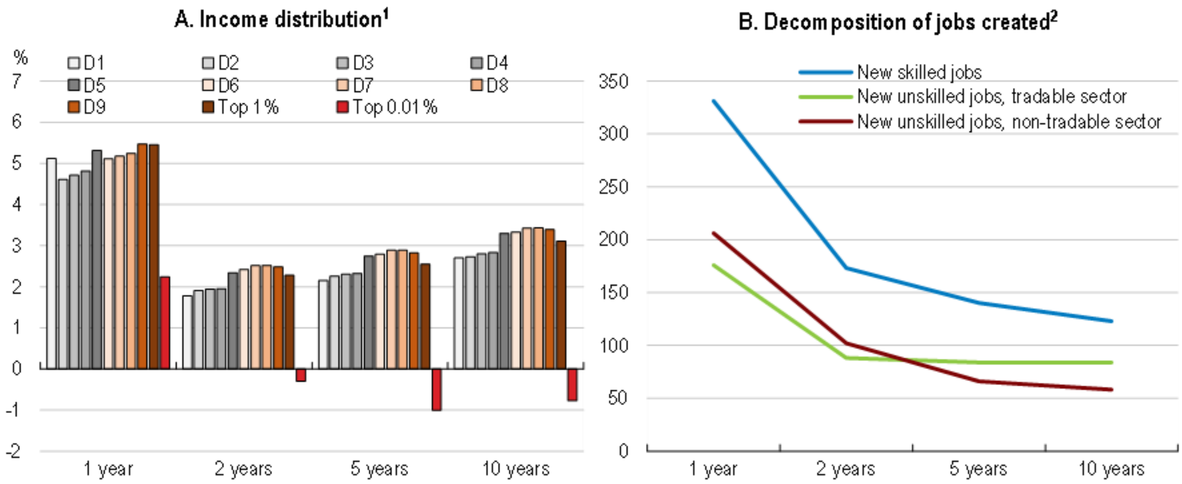
Note: 1. Percentage point deviation from the pre-reform steady state. When the real exchange rate increases, domestic prices decline relative to foreign prices expressed in the local currency.2. In thousands

Figure 7 shows the distributional impact of the product market reform of the tradable sector. Short-run benefits are sizeable, linked to employment growth, and spread across all deciles. The positive impact is lower in the medium to long run but remains positive for all but the very top of the income distribution (top 0.01%), which owns most of the productive capital, which now produces lower profits. As employment and real wage increase, the labour income share rises. However, in the medium to long run, higher income earners, and in particular, skilled workers, benefit more from the reform: Households from the top six deciles, which represent most of workers, and more educated ones, benefit more from the reform than the bottom four deciles, where labour participation is very limited.

Figure 8 shows the distributional impact of the product market reform in the non-tradable sector. The short-run impact on the disposable income is negative for all deciles and in line with the fall in output and declining employment rates. The extreme top of the distribution (the 0.01%) are the only ones losing revenues from the reform, as they own most of the productive capital and suffer from declining profits. Since the value-added of the non-tradable sector is one-thirds of that of the tradable sector, the overall impact on income is low. However, the bargaining power of workers increases, pushing up wages in all sectors, redistributing the benefits of the reform to the whole economy.

Figure 7. The distributional impact of the product market reform in the tradable sector

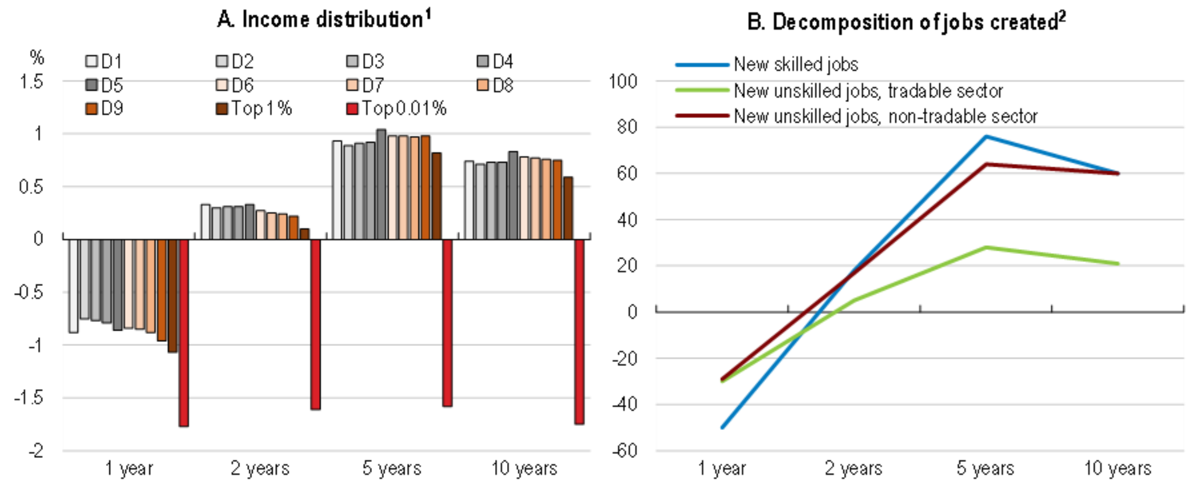
Impact of a reduction of the mark-up by 10%



Note: 1. Percentage point deviation from the pre-reform steady state. D1 stands for decile 1, D2 for decile 2, and so forth. 2. In thousands.

Figure 8. The distributional impact of product market reform in the non-tradable sector

Impact of a reduction of the mark-up by 10%



Note: 1. Percentage point deviation from the pre-reform steady state. D1 stands for decile 1, D2 for decile 2, and so forth. 2. In thousands.

The effect of increasing public investment net of its financing through higher tax

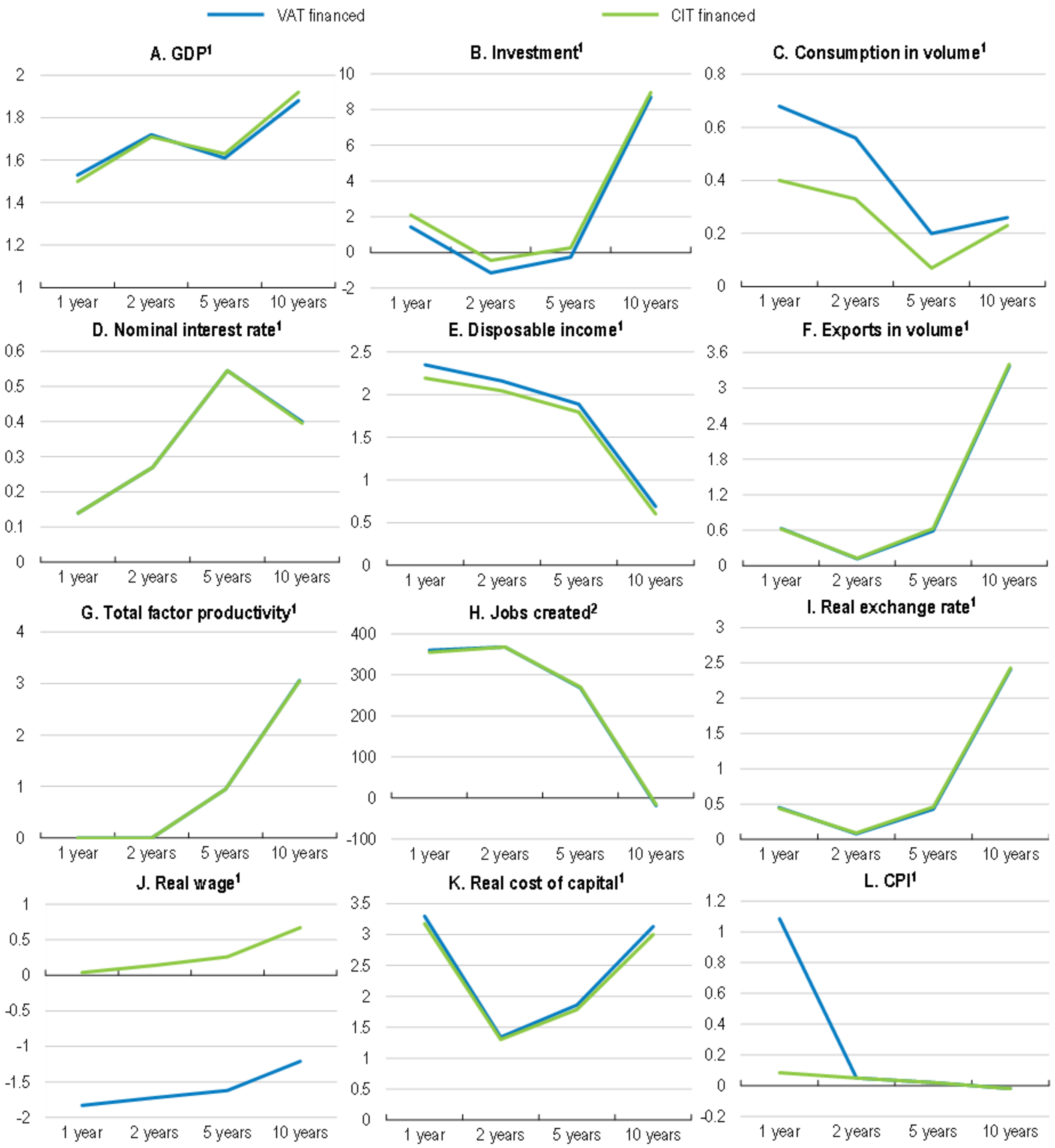
Public investment brings an additional channel in the model as it is linked to the total factor productivity growth. In the baseline model, public investment grows with government revenues and GDP. Most papers incorporating public investment in DSGE models have focused on the multiplier effect of the public investment by analysing the crowding out and crowding in effects (Gali, Lopez-Salido, and Vallés 2007; Christiano et al., 2011; Dupaigne and Fève, 2016). Some papers, as Straub and Tchakarov (2007) and Hickey et al. (2019) analyse explicitly the transitory and long-run effects of public investment by considering its impact through the accumulation of public capital stock. Straub and Tchakarov (2007) analyse the impact of a 1 percent of GDP increase of public investment for the Euro area on growth.

In our model, increasing public infrastructure investment (+ 1 point of GDP) has similar dynamic macroeconomic effects whether financed by raising VAT or capital income taxes (Figure 9). In both cases, the GDP impact is lower than reforming the tradable market in the short run but is higher in the long run. A rise in public infrastructure investment leads to an immediate expansion of output as aggregate demand increases in the short run, which also drives a concurrent positive response of private investment. Thus, in the short run, there is a complementarity between private and public investment under both reform scenarios. However, in the medium run, as the interest rate increases, private investment recedes below its level in the initial steady state. Finally, in the long term, private investment bounces back and expands amid rising total factor productivity, alongside competitiveness gains, which boost exports.

In the short run, consumption increases under both scenarios and remains above its level in the initial steady state. The increase in consumption hinges on two effects. First, investment shock has an immediate effect in aggregate demand, leading to job creations and higher disposable income, though real wages decrease in the case the investment is financed by higher VAT, which substantially ramps inflation up. Second, the investment shock has a supply side effect as it increases total factor productivity in the long term and therefore boosts the marginal productivity of labour and private capital. Even when financed by capital income tax, the public investment shock renders the wealth effect less negative or even positive if the productivity of capital is high enough. This leads to a higher increase in private investment and lower consumption increase when public investment increase is financed by a rise in capital income tax than when it is financed by a rise in VAT.

Figure 9. The impact of increasing public investment financed either by VAT or capital income tax

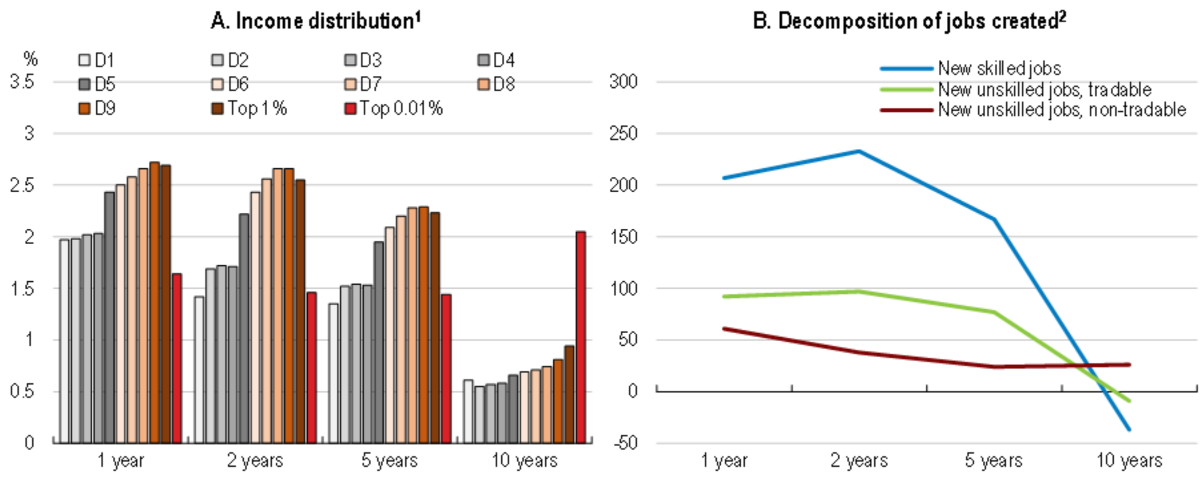
A 1 point of GDP increase in investment



Note: 1. Percentage point deviation from the pre-reform steady state. When the real exchange rate increases, domestic prices decline relative to foreign prices expressed in the local currency. 2. In thousands.

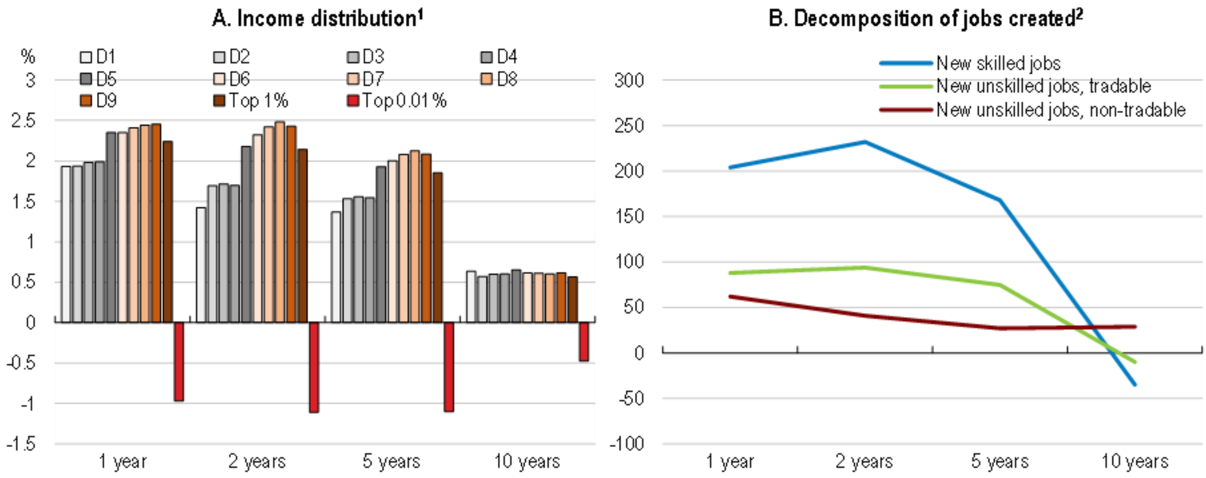
The jobs creation effect of increased public investment is similar whether financed by VAT or capital income tax. Overall, disposable income increases more when VAT is raised than when capital income tax is raised (Figures 10 and 11). One can find here the least distortive property of value-added taxation. In both cases, the top six deciles benefit more from the reform than the bottom four deciles. The top 0.01% benefit from the reform when it is VAT financed while they lose from the reform when it is financed by capital income tax. In the long run, financing the reform by capital income tax is slightly redistributive as bottom deciles benefit more than to top deciles (Figure 10).

Figure 10. The distributional impact of the public investment shock financed by a rise in VAT



Note: 1. Percentage point deviation from the steady state. D1 stands for decile 1, D2 for decile 2, and so forth. The reform is VAT-financed. 2. In thousands.

Figure 11. The distributional impact of the public investment shock financed by a rise in capital income tax



Note: 1. Percentage point deviation from the pre-reform steady state. D1 stands for decile 1, D2 for decile 2, and so forth. The reform is capital income tax financed. 2. In thousands.

Model sensitivity to elasticities

The results underline that price rigidity differs substantially between the tradable and non-tradable sector. Prices tend to be much less rigid in the tradable sector, which is exposed to foreign competition. Retailers of the tradable goods and services can reset 80% of their prices, versus 70% in the non-tradable sector and prices rigidities are very limited. On the contrary, price stickiness is sizeable in the non-tradable sector. Wages are much less sticky than prices in South Africa, about 90% of the wages can be reset each quarter and past inflation tend to play a lower role.

Finally, the fiscal rule is not well identified, which could be related to the existence of three distinct phases of fiscal policy, which has been expansionary between the 1970s, contractionary in the 1980s and expansionary again since the mid-1990s. The long run reaction of government spending to debt is then set at 0.5, meaning that an increase of the debt level by 1% is following by a cut in spending by 0.5% in the long run. Transfers are assumed to be less responsive to activity and the reaction to long-term debt is set to be 0.25.

To display the sensitivity of the model to the elasticities, one can compute the number of jobs created according to the model following standard shocks on: (i) government spending, (ii) foreign demand, (iii) mark-up reduction in the tradable sector and (iv) a general exogenous increase in wages and finally (v) an increase in transfers financed by an increase of the VAT rate. The results are displayed in Table 9 and simulations indicate that the elasticity of substitution between high-skilled and low-skilled labour has only a mild impact in the long run. The elasticity of substitution between tradable and non-tradable services also do not appear to play an import role in the long run. The steady state of the economy is, however, sensitive to the export price elasticity. To a lesser extent, the elasticity of substitution between the domestic and the foreign goods and services also has a substantial impact on the steady state.

Table 9. Sensitivity of long-term response to standard shocks to elasticities

Variable	Baseline	Low - skilled jobs elasticity	High-skilled jobs elasticity	Low export price elasticity	High export price elasticity	Low import price elasticity	High import price elasticity	Low between sectors elasticity	High between sectors elasticity
Government spending	**2**	3	0	3	0	2	1	2	1
Foreign demand	**50**	51	50	60	41	56	43	51	49
Mark-up in tradable sector	**295**	299	292	229	362	252	344	296	292
Wage increase	**-232**	-225	-239	-205	-260	-214	-253	-230	-235
Transfer increase financed by VAT	**-2**	-4	-1	-1	-4	-1	-3	-2	-2
Low-skill/high-skill labour elasticity	**0.5**	1.0	0.25	0.5	0.5	0.5	0.5	0.5	0.5
Export price elasticity	**1.4**	1.4	1.4	1.0	2.0	1.4	1.4	1.4	1.4
η_m	**0.9**	0.9	0.9	0.9	0.9	0.5	1.5	0.9	0.9
η_C	**0.6**	0.6	0.6	0.6	0.6	0.6	0.6	1	0.3

Note: Numbers are the employment impact of the different variant compared to the baseline. The column title indicates the parameter which is changed in combination with policy variables in the first 5 rows.

Conclusion

This paper quantitatively assesses policy choices between reforming the product market and boosting public investment. Reducing mark-ups in tradable product markets are found to have larger benefits in the short run than boosting public infrastructure investment. However, the latter induces a larger increase in GDP through total factor productivity growth in the long run. In contrast, product market reforms in the tradable goods sector have higher and more persistent employment effects than boosting public investment. In particular, product market reforms allow efficiency gains and capital reallocation that favour job creation.

This paper also reconciles a host of findings in the literature on the impacts of product market reforms. Specifically, past findings on the short-term recessionary impacts of product market reforms are confirmed, especially for reforms that target the non-tradable sector (Cacciatore and Fiori, 2016). In contrast, a short-term expansionary impact is found when the reform targets the tradable sector (Alvés et al., 2017). For future research avenues, this model could be employed to analyse labour market reforms and extended to feature corporate income taxes to assess the effects of structural reforms on the cost of capital and private investment in greater detail.

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Technical appendix on derivations of the model

One can use generic log-linearization results for a CES aggregate *Z* with two elements *X* and *Y* the elasticity of substitution being σ. Let us denote for that purpose α the share of the first component in value at the steady state. The aggregate expressed as deviation in logarithm to the steady state (*z*) follows the deviations in the elements, *x* and *y*.

$$z = \alpha x + (1 - \alpha)y$$

The relative price of the aggregate q_z can be computed as well from the price of the components, q_x and q_y:

$$q_z = \alpha q_x + (1 - \alpha)q_y$$

This allows to express the components from the aggregate and the prices:

$$x = z + \sigma(q_z - q_x)$$

To simplify, we define the change in tax rates as follows, with τ^* the value of the tax rate at the initial steady state:

$$\Delta\tau = \frac{\tau - \tau^*}{1 + \tau^*}$$

The input price is derived from the minimization of the cost of the aggregate. The price (in levels) of the composite of tradable goods Q in sector *j* is given by the following, where $Q^\star$ is the relative price of the imported good in domestic currency to the production price of the tradable good (which equal the terms of trades here). The real tax rate of domestic inputs is denoted τ_1^z, while the real tax rate of imported inputs (which include tariffs) is denoted τ_m^z. The price of the aggregate of imported and domestic tradable input is denoted Q_v^j

$$\left(Q_v^j\right)^{1-\eta_v} = \left(\kappa_v^j\right)^{\eta_v}(1 + \tau_1^z)^{1-\eta_{vz}} + \left(1 - \kappa_v^j\right)^{\eta_v}\left((1 + \tau_m^z)Q^\star\right)^{1-\eta_v}$$

The final input price faced by firms in sector *j* is then Q_z^j:

$$\left(Q_z^j\right)^{1-\eta_z} = \left(\kappa_z^j\right)^{\eta_z}\left(Q_v^j\right)^{1-\eta_z} + \left(1 - \kappa_z^j\right)^{\eta_z}(Q_2)^{1-\eta_z}$$

The demand of the three types of inputs in the three sectors *j* is given by the following equation where M_z^j is the volume of imported intermediate input for use in sector *j*:

$$Z_2^j = \psi_j Y_j\left(1 - \kappa_z^j\right)^{\eta_z}\left(Q_z^j\right)^{\eta_z}\left((1 + \tau_2^z)Q_2\right)^{-\eta_z}$$

$$Z_1^j = \psi_j Y_j(1 + \tau_1^z)^{-\eta_v}\left(\kappa_v^j\right)^{\eta_v}\left(\kappa_z^j\right)^{\eta_z}\left(Q_{zj}\right)^{\eta_z}\left(\kappa_v^j\right)^{\eta_v - \eta_z}$$

$$M_z^j = \psi_j Y_j(1 + \tau_1^z)^{-\eta_v}\left(1 - \kappa_v^j\right)^{\eta_v}\left(\kappa_z^j\right)^{\eta_z}\left(Q_z^j\right)^{\eta_z}\left(Q_v^j\right)^{\eta_v - \eta_z}(Q^\star)^{-\eta_v}$$

The demand can be log-linearized easily using the properties of the CES functions:

$$z_2^j = y^j + \eta_z\left(q_z^j - q_2 - \Delta\tau_2^z\right)$$

$$z_1^j = y^j - \eta_v\Delta\tau_1^z + \eta_z q_z^j$$

$$m_z^j = y^j - \eta_v \Delta\tau_1^z + \eta_z q_z^j + (\eta_v - \eta_z) q_v^j - \eta_v (q^\star + \Delta\tau_\star^z)$$

Consumption

The level of imported goods for Ricardian households is given by:

$$C_1^o = M_c^o \left(\frac{\kappa_o^u}{1-\kappa_o^u}\right)^{\eta_u} \left(\frac{1+\tau_m^c}{1+\tau_1^c}\right)^{\eta_u} (Q^\star)^{\eta_u}$$

The consumption of domestic services can also be deduced:

$$C_2^o = M_c^o \left(\frac{\kappa_o^c}{1-\kappa_o^c}\right)^{-\eta_c} (1-\kappa_o^u)^{-\eta_u} \frac{(1+\tau_m^c)^{\eta_u}}{(1+\tau_2^c)^{\eta_c}} (Q_2)^{\eta_c} (Q_u^o)^{\eta_c-\eta_u} (Q^\star)^{\eta_u}$$

The optimal consumption of the constrained agent becomes:

$$C_2^c = Y_c (1-\kappa_c^c)^{\eta_c} (q_c^c)^{\eta_c} ((1+\tau_2^c) Q_2)^{-\eta_c}$$
$$C_1^c = Y_c (1+\tau_1^c)^{-\eta_u} (\kappa_c^u)^{\eta_u} (\kappa_c^c)^{\eta_c} (Q_c^c)^{\eta_c} (Q_u^c)^{\eta_u-\eta_c}$$
$$M_c^c = Y_c (1+\tau_1^c)^{-\eta_u} (1-\kappa_c^u)^{\eta_u} (\kappa_c^c)^{\eta_c} (Q_c^c)^{\eta_c} (Q_u^c)^{\eta_u-\eta_c} (Q^\star)^{-\eta_u}$$

Private investment

The investments are proportionate to the aggregate investments:

$$Inv_2^j = Inv^j (1-\kappa_i)^{\eta_i} (Q_i)^{\eta_i} \left((1+\tau_2^i) Q_2\right)^{-\eta_i}$$

$$Inv_1^j = Inv^j \left(1+\tau_1^i\right)^{-\eta_v} (\kappa_i^v)^{\eta_v} (\kappa_i)^{\eta_i} (Q_i)^{\eta_i} (Q_i^v)^{\eta_v-\eta_i}$$

$$M_i^j = Inv^j \left(1+\tau_1^i\right)^{-\eta_v} (1-\kappa_i^v)^{\eta_v} (\kappa_i)^{\eta_i} (Q_i)^{i} (Q_i^v)^{\eta_v-\eta_i} (Q^\star)^{-\eta_v}$$

Derivation of equation (47).

We start from the definition of the terms of trade:

$$Q_t = \frac{E_t P_t^*}{P_t}$$

We log-linearize this equation and we take its increase over one period:

$$q_t - q_{t-1} = e_t - e_{t-1} + p_t^\star - p_{t-1}^\star - (p_t - p_{t-1})$$

Recognizing that at the first order: $\pi_t \approx p_t - p_{t-1}$ and $\pi_t^\star \approx p_t^\star - p_{t-1}^\star$, this gives equation (47).